LUISS Guido Carli
School of European Political Economy

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Pierpaolo Benigno

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A Central Bank Theory of Price Level Determination*

Pierpaolo Benigno
LUISS and EIEF

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Abstract

A theory in which the central bank controls the price level is put forward as an alternative to the fiscal theory of the price level. It is not necessary to have a fiscal stimulus to avoid liquidity traps nor a fiscal anchor to disallow inflationary spirals. A central bank appropriately capitalized can succeed to control the price level by setting the interest rate on reserves, holding risk-free assets and rebating its income to the treasury – from which it has to maintain financial independence. If the central bank undertakes unconventional open-market operations, either it has to give up its financial independence or leaves the economy exposed to self-fulfilling inflationary spirals or chronic liquidity traps.

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1 Introduction

The determination of prices has been at the center of the economic debate since the existence of monetary systems with discussions ranging from the policies that monetary institutions should follow to the assets that they should hold to back the value of money.

The instability and volatility associated with environments pervaded by multiple equilibria can undermine the role of the central bank in achieving inflation goals. A recent literature has taken the view that fiscal policy is not less important than monetary policy in determining the price level, and that some degree of 'activism' on the actions of the fiscal authority is needed to appropriately back the price level (see among others Cochrane, 2011). In his Presidential Address at the American Economic Association, Sims (2013) has pointed out that the literature on the fiscal theory of the price level has at the end recognized that "fiscal policy can be a determinant, or even the sole determinant, of the price level."

Absent the backing of fiscal policy, inflationary or deflationary spirals can develop leaving the central bank completely helpless. To rule out deflations, fiscal policy should be substantially stimulative coupled with an appropriate expansion of central bank's liabilities. To disallow inflationary spirals, the fiscal authority should exercise in an effective way its taxation capacity to be able to anchor the inflation rate at the desired target.

Following this view, it is often argued that the architecture of the European Monetary Union is established on precarious foundations since the monetary authority does not have a direct fiscal authority behind it, while the many national tax authorities are constrained to follow strict requirements on their budget policies.[1]

This work challenges the above view showing that fiscal 'activism' is not necessary to control the price level and offers an alternative perspective according to which the central bank achieves the control of the price level by relying only on its own means.

There are some salient features of the proposal. At its inception, the central bank receives an appropriate capitalization from the treasury and borrows additional resources through interest bearing securities (reserves) or money. Its portfolio of assets consists only of short-term riskless bonds. Monetary policy is specified by setting the interest rate on reserves which

[1] See Sims (1999, 2016).

actively reacts to the deviations of prices (or inflation) with respect to the target. Central bank's profits are remitted to the treasury, and then to the private sector. Finally, the central bank has to be financially independent from the treasury.

All of the above features, which to a certain extent are not far from current central-banking practices, are sufficient to allow the central bank to control the price level ruling out deflationary and inflationary spirals.[2] Regarding financial independence, this is a two-sided concept. On the one side, it means that any attempt from third parties to obtain extraordinary dividends or deplete central bank's resources should be ruled out.[3] On the other side, the central bank should not receive further treasury's support beyond the initial capitalization.[4]

The reason for why monetary policy alone can control the price level depends on two important observations. First, in a fiat monetary system, the central bank's liabilities have a special role since they define the 'unit of account', and by this virtue they are free of any risk.[5] Differently from any other agent in the economy, the central bank is not subject to a solvency condition or exposed to run. This property gives the central bank special powers, in particular when ruling out inflationary spirals. Second, any monetary policy action has 'fiscal' consequences thereby implying transfers to the treasury and then to the private sector. Whereas the central bank can issue its liabilities at will regardless of solvency issues, solvency together with the composition of the balance sheet and the remittances' policies becomes instead important in determining the value of those liabilities in terms of

[2] In the case of inflationary solutions, the remittances' policy should switch (or at least threaten to switch) to a real dividend policy that anchors the value of central bank's net worth at the target price level.

[3] To see how far we are in some cases from the concept of financial independence suggested by the proposal, consider the recent FAST Act, enacted on December 4, 2015, requiring that aggregate Federal Reserve Bank capital surplus not exceed $10 billion, which resulted in a transfer of $19.3 billion from the Federal Reserve Bank to the Treasury. I am grateful to one anonymous referee for pointing this out. See also the discussion of Buiter (2009) who emphasizes, instead, that the financial independence of the European Central Bank is higher than any other central bank since it does not have a single tax authority behind it.

[4] This second side is indeed achieved by the riskless composition of central bank's assets while the first side should be part of the features that needs to be set at the central bank's inception.

[5] See Woodford (2000, 2001a). I am indebted to Michael Woodford for insightful conversations on this point.

goods – the inverse of the price level.

To rule out deflations or liquidity traps, it is sufficient that the central bank is committed to keep constant the value of its nominal net worth. Financial independence is important to this end. By maintaining nominal net worth constant, even in a deflation, the central bank retains in its balance sheet an amount of real resources that does not vanish over time and actually grows. This is exactly the reason why deflations cannot form: the goods market does not clear at deflationary prices – there is an excess supply of goods over demand – unless those central bank's resources are fully expropriated and rebated to the private sector to close the shortage in goods demand. Therefore, the policy prescription to rule out deflations is to have a financially-independent central bank that is shielded from any third-party raid on its net worth.

To disallow inflationary spirals, it is again important to maintain a positive value of central bank's nominal net worth but this is now key to back the price level. The central bank can commit to sell shares of its nominal equity and promise a stream of real dividends that anchors the price level at the desired target via a no-arbitrage condition. In this case, the ability of the central bank to transfer a certain amount of real resources to the private sector relies on the special power of its liabilities that can be increased at will without any solvency problem.[6]

What is the role of the treasury in this picture, beyond the initial capitalization? The treasury is treated as any other borrower that needs to 'passively' adjust its real primary surplus or deficit to meet its obligations, if they are risk free, or, otherwise, seizes them.

At the end, the analysis shows that even the architecture of the European Monetary Union, with many tax authorities constrained by budget rules and no a single authority directly behind the central bank, does not jeopardize the control of the price level by the European Central Bank.

All the features described above are important for the results. If the central bank does not receive initial capital, while maintaining the other elements unchanged, inflationary and deflationary solutions can develop. Similarly, if the central bank purchases risky securities it has either to give up financial independence or lose full control of the price level.

[6]An alternative approach discussed in the paper that does not entail an increasing path of nominal liabilities imposes instead an implicit tax on the private sector by setting reserve requirements. This taxation ability is in the powers of the central bank because its liabilities are the ultimate settlements of all payments in the 'unit of account'.

This paper is related to an important literature that has discussed the issue of price determination in general equilibrium monetary economies ranging from the fiscal theory of the price level as in Cochrane (2001), Leeper (1991), Sims (1994, 2000, 2013), Woodford (1995, 2001) to theories of price determination through active interest rate rules supported by fiscal backing as in Benhabib et al. (2001, 2002), Schmitt-Grohé and Uribe (2000), Sims (2013), Woodford (2003).[7] Cochrane (2011) provides an extensive and critical discussion of results of determinacy achieved through Taylor's rules.

With respect to all this literature, the contribution of this work is to emphasize that the determination of the price level can be left to the central bank without any fiscal backing or support resting on an appropriate design of how central banks should operate starting from their capital, composition of assets, remittances' policy and policy rule. One of the main insights of this work stands on the separation between the budget constraint of the treasury and that of the central bank, as suggested by a recent literature following Bassetto and Messer (2013), Benigno and Nisticò (2015), Berriel and Bhattarai (2009), Del Negro and Sims (2015), Hall and Reis (2015), Reis (2015), Sims (2000, 2005). However, the main difference is that this literature is not concerned about a global analysis of the determination of the price level.

There are some works in the literature that share the same absence of fiscal 'activism' in the determination of the price level. Obstfeld and Rogoff (1983) have shown that deflationary solutions can be ruled out by targeting the growth of money supply while inflation can be stopped by backing money with a commodity.[8] In their work, the central bank controls money supply while, here, the central bank sets its policy in terms of the nominal interest rate and the economy can be even cashless. To rule out inflationary solutions, Woodford (2001b, 2003 ch. 2.4) proposes an interest rate rule that implies an infinite reaction at a positive inflation rate. Similarly, but through a different mechanism, one of the solutions of this paper to prevent inflationary spirals implies a threat to blow up inflation. On the other side, when trimming deflationary solutions, Woodford (2001b, 2003 ch. 2.4) relies on fiscal 'activism' in contrast to what proposed in this work. Bassetto (2004) shows that the central bank can disallow deflationary solutions by imposing

[7] For an empirical evaluation see among others Canzoneri et al. (2001) and Bianchi and Ilut (2017).

[8] Cochrane (2011) discusses how the solution of Obstfeld and Rogoff (1983) does not lead to uniqueness of equilibria but just stop inflationary paths.

negative nominal interest rates as a way to reflate the economy by creating arbitrage opportunities. Hall and Reis (2016) have suggested that the central bank should control the price level by committing to a real payment policy on reserves in place of using a standard policy rule on the nominal interest rate. Instead, this work maintains the use of a nominal interest rate policy and further exploits the fiscal implications of alternative specifications of central bank's balance sheets.

The structure of the paper is the following. Section 2 presents a simple monetary model. Section 3 discusses the fiscal theory of the price level. Section 4 presents the proposal of this work. Section 5 extends the model to include long-term securities and a non-pecuniary value of money balances. It also discusses the implications for price determination of considering risky open-market operations in absence of treasury's support. Section 6 concludes.

2 The problem of price level determination

To discuss my argument in a consistent way with the literature I follow Cochrane (2011) who presents a simple endowment monetary economy featuring two agents, the consumer and the government.

The monetary economy is characterized by a currency, let's say dollars, that serves as a 'unit of account' and 'store of value'. Both properties are important for the analysis that follows. Let me first focus on what a 'unit of account' means and its implications. On the one side, a 'unit of account' is the unit of measure to value goods and securities, the *numéraire*. In this simple monetary economy there is only one 'unit of account' and the price of all goods and securities are quoted in that 'unit of account'. On the other side, a fiat 'unit of account' is the liability of an agent (and only of one agent) which in the model is the central bank. By this virtue, the price of one unit of central bank's liability is just one dollar because that unit of liability exactly defines what a dollar is – a concept extensively discussed by Woodford (2000, 2001a).[9] Therefore, one dollar claim at the central bank is risk-free regardless of the resources that the central bank has and its balance-sheet composition.[10]

[9] Sandroni (2006) has reconnected Woodford's analysis to Kaldor (1980)'s view of money that "rules the roost".

[10] This property is not shared by any other agent in the economy since their liabilities are denominated in the 'unit of account', but do not define the 'unit of account'. A dollar

I am going to assume that the central bank can issue its liabilities in two different ways: i) money, i.e. banknotes or coins which are physical means of payments and ii) reserves, which are one-period short-term securities. Moreover, the central bank sets its monetary policy by paying an interest rate, i_t^R, on reserves. Since reserves define the 'unit of account', the central bank can set their interest rate independently of the quantity issued.[11] By setting i_t^R the central bank is also determining the short-term interest rate, i_t, on any other riskless security issued in the economy. Absence of arbitrage opportunities implies that $i_t = i_t^R$. In what follows, I am going to simply use the notation i_t in a interchangeable way to denote either the interest rate on reserves or that on other risk-free securities.

Let me now focus on the property of currency as a 'store of value' and its implications. For its physical characteristics, money serves as a store of value. The existence of money implies that the interest rate on reserves cannot be negative, otherwise arbitrage opportunities would arise. In the simple model of this section, I am assuming that money and reserves provide the same payment services. Therefore the demand of money is going to be zero whenever the interest rate on reserves is positive. When, instead, the interest rate on reserves is zero, money and reserves are perfect substitute. Without losing generality, I am setting the demand of money to zero even in this case. Therefore, as in other papers, I am at end modelling a completely cashless economy.[12]

One important difference with respect to Cochrane (2011) is that I am going to assume that the monetary system starts at time t_0, which implies that the economy does not inherit any security denominated in dollars from period $t_0 - 1$. This environment serves the purpose of studying whether it is possible to design from scratch institutions that can control the price level, without any inheritance from the past.

I leave the details of the model to the Appendix. I will restrict my attention to a perfect-foresight equilibrium. In a constant-endowment economy, the Euler equation implies a relationship between the nominal interest rate

debt issued by these agents is therefore priced at the market value.

[11] See again Woodford (2000, 2001a).

[12] Note that the economy can also be completely reserve-less without this being a problem for the control of the interest rate i_t^R since "clearing balances at the central bank will still define the thing to which these other claims are accepted as equivalent" (Woodford, 2000) even in a world in which the demand or supply of clearing balances (reserves) at the central bank is zero.

and the inflation rate, through the real rate,

$$1 + i_t = \frac{1}{\beta}\frac{P_{t+1}}{P_t}, \tag{1}$$

where β (with $0 < \beta < 1$) is the rate at which the consumers discount consumption – its inverse is equal to the gross real rate; P_t is the price level. To get (1), I have used equilibrium in the goods market, $c_t = y$ at each date t where c_t is real consumption and y is the constant endowment.

The nominal interest rate on reserves is set by the central bank to follow the simple rule

$$1 + i_t = \max\left\{\frac{1}{\beta}\left(\frac{P_t}{P^*}\right)^{\phi}, 1\right\} \tag{2}$$

where ϕ is a non-negative parameter, $\phi \geq 0$; P^* is positive with $P^* > 0$. I am assuming that at its inception the central bank receives a mandate in terms of the target price level, P^*.[13] When $\phi > 0$ the instrument of policy reacts directly to the deviation of the actual price level with respect to the target. When $\phi = 0$, the nominal interest rate is pegged to a constant value, the real rate, but P^* is still the objective of policy.[14]

Combining (1) and (2), the price level follows a non-linear difference equation:

$$\frac{P_{t+1}}{P_t} = \max\left\{\left(\frac{P_t}{P^*}\right)^{\phi}, \beta\right\}. \tag{3}$$

Equation (3) has infinite solutions irrespective of the value $\phi \geq 0$. Consider first the case $\phi > 0$ which is shown in Figure 1. There is a stationary solution, with $P_t > 0$, if and only if $P_{t_0} = P^*$. If instead $P_{t_0} > P^*$, the solution will be monotone increasing, an inflationary solution. On the other side, if $P_{t_0} < P^*$, the solution will be monotone decreasing, a deflationary solution, and in particular when $P_t \leq \beta^{1/\phi}P^*$ the rate of deflation is β. Note, moreover that solutions associated with different P_{t_0} never cross along the

[13]This target can be also self-imposed by the central bank.

[14]By assuming the policy rule (2), I am departing from the assumption of Cochrane (2011), and many others, in which the nominal interest rate reacts to the deviations of current inflation rate with respect to a target. Were this the case, indeed, price determination would inherit an initial condition, namely the price level at time $t_0 - 1$ which is not defined in my framework since the monetary system starts at time t_0. Although I could overcome the problem by arbitrarily fixing P_{t_0-1} at any fictitious positive number, by assuming the rule (2) I completely avoid the issue without losing any generality at all.

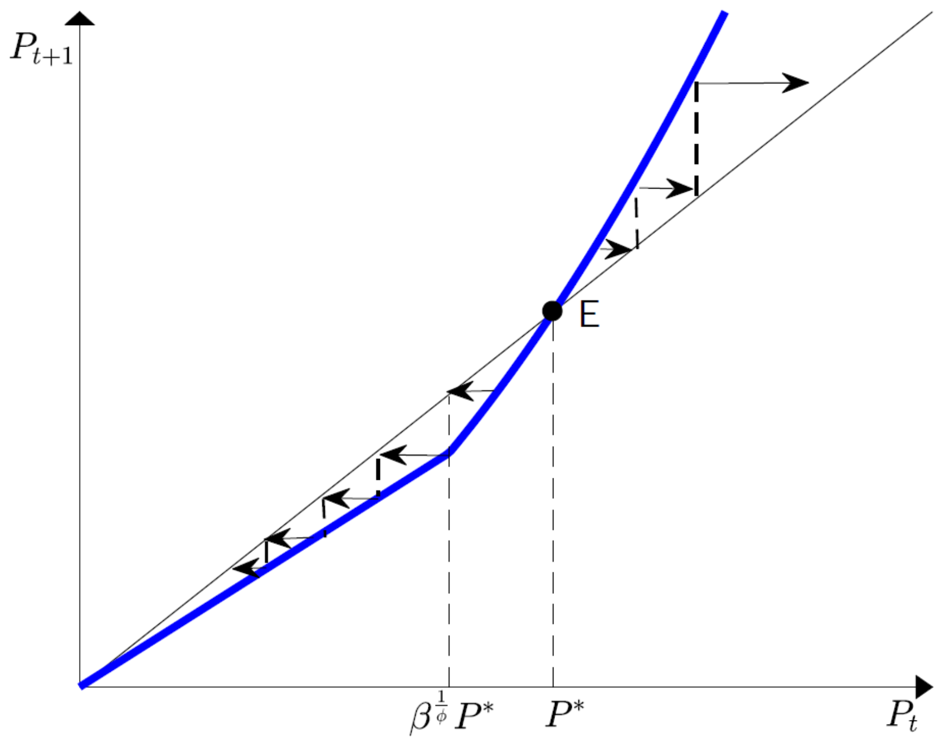

Figure 1: Plot of the difference equation (3) in which $\phi > 0$. The point E is the stationary solution $P_t = P^*$ at each date t. If $P_t \leq \beta^{\frac{1}{\phi}} P^*$ the rate of deflation of the price level, P, is β, with $0 < \beta < 1$.

time dimension. Therefore when $\phi > 0$, there are infinite solutions which can be simply indexed by the value taken by the initial price level, P_{t_0}, in the interval $(0, \infty)$.

Consider now the case $\phi = 0$. Infinite solutions are also possible which can also be indexed by the value taken by the initial price level P_{t_0} in the range $(0, \infty)$. However, all these solutions are stationary.

In this simple framework, the problem of price level determination is that of nailing down the price level at time t_0 possibly at the target P^*. Before describing my proposal, let me first address the solution offered by the fiscal theory of the price level.

3 Fiscal theory of the price level

The key insight of the fiscal theory of the price level is that other equilibrium conditions should be exploited to uniquely determine the price level. As a matter of fact, I have only characterized the solutions of (3) but not equilibria. To this end, I have to enrich the presentation of the model.

Consumers are maximizing intertemporal utility starting from period t_0. Intertemporal utility is separable with a discount factor given by β and utility flow $u(c_t)$, where $u(\cdot)$ is a concave function. Consumers face the following budget constraint:

$$\frac{B_t}{1+i_t} = B_{t-1} + P_t(y - c_t) - P_t\tau_t. \tag{4}$$

They can lend or borrow using short-term riskless securities, B_t, at the interest rate i_t (a positive B_t indicates assets); τ_t are lump-sum real taxes levied by the treasury net of transfers, i.e. the real primary surplus.[15] The consumer's problem is subject to a natural borrowing limit at each time $t \geq t_0$

$$\frac{B_t}{P_{t+1}} \geq -\sum_{j=0}^{\infty} R_{t+1,t+1+j}(y - \tau_{t+1+j}) > -\infty \tag{5}$$

saying that the debt to be paid at a generic time $t+1$, and contracted at time t, cannot exceed in real term the present-discounted value of real net income, where $R_{t+1,t+j}$ is the appropriate market discount factor to evaluate

[15] Given that the economy is cashless, as I have already explained, I am completely abstracting from money in the writing of the budget constraint.

a unit of good at time $t+j$ with respect to time $t+1$, with $R_{t+1,t+1} \equiv 1$. The optimization problem of the consumer implies that the Euler equation holds

$$u(c_t) = \beta(1+i_t)\frac{P_t}{P_{t+1}}u(c_{t+1}) \tag{6}$$

in an interior solution for each $t \geq t_0$ and that the consumer exhausts his intertemporal budget constraint:

$$\sum_{t=t_0}^{\infty} R_{t_0,t}c_t = \sum_{t=t_0}^{\infty} R_{t_0,t}(y-\tau_t). \tag{7}$$

The present-discounted value of consumption should be equal to the present-discounted value of net income. In the above intertemporal budget constraint there is no financial wealth carried from period $t_0 - 1$, since I have assumed that the monetary system starts at period t_0 and therefore $B_{t_0-1} = 0$.

The mirror image of the exhaustion of the intertemporal budget constraint of the consumer is the transversality condition

$$\lim_{t\to\infty}\left\{R_{t_0,t}\frac{B_t}{P_t(1+i_t)}\right\} = 0, \tag{8}$$

that constraints the long-run behavior of the assets (or debt) held by the consumer.[16]

In what follows I simplify the analysis to log utility, that is $u(c_t) = \ln c_t$. After substituting the set of Euler equations into the intertemporal budget constraint, I get

$$c_{t_0} = (1-\beta)\left\{\sum_{t=t_0}^{\infty} R_{t_0,t}(y-\tau_t)\right\}, \tag{9}$$

which represents the demand of consumption goods at time t_0 given the present-discounted value of the real net income of the consumer.

Equilibrium in the goods market, $c_t = y$, implies that the discount factor $R_{t_0,t}$ is equal to $R_{t_0,t} = \beta^{t-t_0}$ and that either (7) or (8) imply that in equilibrium the following intertemporal budget constraint of the government holds:

[16] Equations (7) and (8) are equivalent equilibrium conditions taking into account (4) and the initial condition $B_{t_0-1} = 0$.

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \tau_t = 0. \tag{10}$$

The present-discounted value of real primary surplus should be equal to zero. This is the key equilibrium condition on which the fiscal theory of the price level rests to determine prices, although it is not in the standard form seen in the literature because there is no outstanding debt as of time $t_0 - 1$. However, I am going to show that this is not necessary to determine the price level, even though some government nominal liabilities should be outstanding at some point in time.[17]

The main insight of the fiscal theory of the price level is that (10) can determine prices because it holds for equilibrium price sequences and not necessarily for all price sequences that solve (3). Specularly, price sequences solving (3) can be ruled out as equilibria if they imply violations of (10).

To determine equilibrium prices, consider the following specification of the path of real primary surpluses $\{\tau_t\}_{t=t_0}^{\infty}$. Let the government run a deficit at time t_0 in real terms, $\tau_{t_0} = \tau_{t_0}^* < 0$, and instead set the path of future real primary surpluses $\{\tau_t\}_{t=t_0+1}^{\infty}$ at the level $\tau_t = \tau_t^*$ under the following restriction

$$\sum_{t=t_0+1}^{\infty} \beta^{t-t_0-1} \tau_t^* = \frac{B_{t_0}^G}{P^*}. \tag{11}$$

The discounted path of real primary surpluses, as of time t_0+1, is independent of the price level at the same time but directly related to the outstanding nominal liabilities, $B_{t_0}^G$, that the government has to pay at time $t_0 + 1$. Use (11) and $\tau_{t_0} = \tau_{t_0}^*$ into (10) to obtain

$$\beta \frac{B_{t_0}^G}{P^*} + \tau_t^* = 0. \tag{12}$$

Consider now the government's flow budget constraint

$$\frac{B_t^G}{1 + i_t} = B_{t-1}^G - P_t \tau_t, \tag{13}$$

where B_t^G is government debt with initial condition $B_{t_0-1}^G = 0$. In equilibrium $B_t^G = B_t$. Since $\tau_{t_0} = \tau_{t_0}^*$, the budget constraint (13) implies that the

[17] Niepelt (2004) has argued that without oustanding nominal government debt issued at time $t_0 - 1$, the price level would be indeterminate unless fiscal policy imposes an appropriate combination of nominal and real transfers.

government has to issue debt in the amount $B^G_{t_0} = -(1 + i_{t_0})P_{t_0}\tau^*_{t_0}$ which in turn implies that $\tau^*_{t_0} = -\beta B^G_{t_0}/P_{t_0+1}$, having used the Fisher equation (1).

After substituting $\tau^*_{t_0} = -\beta B^G_{t_0}/P_{t_0+1}$ into (12), I obtain

$$\beta\left(\frac{B^G_{t_0}}{P^*} - \frac{B^G_{t_0}}{P_{t_0+1}}\right) = 0. \tag{14}$$

The above equation is satisfied if and only $P_{t_0+1} = P^*$. Therefore, if the government commits to $\{\tau^*_t\}^{\infty}_{t=t_0}$, there is only one equilibrium path of prices: $P_t = P^*$ for each $t \geq t_0$.[18]

To get the result from a different perspective, I can use (9) at the equilibrium discount factor $R_{t_0,t} = \beta^{t-t_0}$, to obtain

$$c_{t_0} = y - (1-\beta)\sum_{t=t_0}^{\infty}\beta^{t-t_0}\tau_t.$$

Now, substitute the specification of fiscal policy $\{\tau^*_t\}^{\infty}_{t=t_0}$ for the sequence $\{\tau_t\}^{\infty}_{t=t_0}$ to get

$$c_{t_0} = y - (1-\beta)\beta\left(\frac{B^G_{t_0}}{P^*} - \frac{B^G_{t_0}}{P_{t_0+1}}\right)$$

which can be further written as

$$c_{t_0} = y + (1-\beta)\tau^*_{t_0}\left(\frac{P_{t_0}}{P^*}\cdot\max\left\{\left(\frac{P_{t_0}}{P^*}\right)^{\phi}, \beta\right\} - 1\right),$$

having used $B^G_{t_0} = -(1 + i_{t_0})P_{t_0}\tau^*_{t_0}$ and equations (1) and (3). The above equation represents the demand of goods at time t_0 conditional on the fiscal policy regime. Goods market at time t_0 can only clear whenever $P_{t_0} = P^*$. If prices were higher than P^* then there would be excess supply of goods ($c_{t_0} < y$) given that $\tau^*_{t_0} < 0$ and prices would fall. If prices were lower then P^*, the excess demand of goods ($c_{t_0} > y$) would bring them back to the target. Note that all these reasonings apply regardless of the value taken by ϕ, with $\phi \geq 0$. In the above characterization, I have assumed that fiscal policy runs a deficit for one period and then backs the price level using the present-discounted value of real primary surpluses starting from the next period. The analysis can be replicated even if the deficits are run for longer horizon and the backing postponed.

[18] Note, however, that a barter economy is always an equilibrium.

3.1 The credibility of government commitment

The fiscal theory of the price level determines uniquely the path of prices by ruling out inflationary and deflationary solutions via the mechanism described above. In this section, I am going to underline the importance of a key feature of the notion of competitive equilibrium used, i.e. that the sequence $\{\tau_t\}_{t=t_0}^{\infty}$ is taken as given by the consumers when maximizing utility under the constraints (4) and (5). In particular, consumers understand that the sequence $\{\tau_t\}_{t=t_0+1}^{\infty}$ satisfies (11) without questioning the credibility of the commitment. This is coherent with the notion of competitive equilibrium since, indeed, $\{\tau_t^*\}_{t=t_0}^{\infty}$ is what observed in equilibrium. However, as shown in the previous section, the beliefs of the consumers on the path followed by fiscal policy are critical to rule out deviations of P_{t_0} from P^* already at time t_0. To be clear, if the price level at time t_0 is lower than P^*, the adjustment mechanism operating through the excess demand of goods, which pushes up the price level, relies on the consumer belief that the tax policy (11) is going to be implemented in such circumstances. If the tax policy is unfeasible or not going to be implemented in these conditions, for reasons that I will explain, then prices below P^* are equilibrium prices. Therefore to prove uniqueness of the equilibrium, it is important to investigate whether the commitment taken by the government is credible enough, i.e. even pursued under conditions that are not observed in the desired equilibrium.[19]

In what follows, whenever I am analyzing deflationary or inflationary solutions, I am implicitly focusing on the case in which $\phi > 0$ in the policy rule (2). But the analysis applies also to the case $\phi = 0$ with appropriate amendments.

Suppose that a deflationary path develops and that $P_t < P^*$ at a generic time t. The government reaches period t with outstanding real debt B_{t-1}/P_t but the commitment (11) promises a path of real primary surplus that is below the outstanding level of obligations that the government would face at that time

$$\sum_{T=t}^{\infty} \beta^{T-t} \tau_T^* = \frac{B_{t-1}}{P^*} < \frac{B_{t-1}}{P_t}. \tag{15}$$

This is indeed consistent with the proposal of Benhabib et al. (2001), namely that, in a deflation, the government should commit to reduce taxes in order

[19] Bassetto (2002) has analyzed the equilibrium that would result as an outcome of a strategic game between government and private sector.

to inflate the economy. However, as shown by the above equation, the commitment leaves part of the outstanding real obligations unbacked.

To understand the credibility of this commitment, I have to go back to the notion of a 'unit of account' that I have explained in the previous section. Is B_{t-1} just riskless debt denominated in the 'unit of account' or is part of the liabilities that define the 'unit of account'? And in the former case, is there a connection, even at some future point in time, with the liabilities that define the 'unit of account'?

To clarify these questions, I provide a simple example. Consider the European Monetary Union, in which the 'unit of account' is defined in terms of the liabilities of the European Central Bank. As I explained, the ECB can set its policy by fixing the interest rate on reserves even if it stands ready to supply zero reserves. Suppose it does. In this case, B does not denote the ECB's liabilities but sovereign debt, denominated in euro, of a group of countries belonging to the union. The above inequality shows that, in a deflation, the real value of this debt is less than the resources the countries commit to pay. Three things can happen: i) the debt remains risk free; ii) it is defaulted on; iii) it is fully backed by central bank's reserves. In the first case, countries have to increase their real primary surpluses to back all the outstanding real obligations. It follows that deflations are going to be equilibria since the initial commitment is not credible at all. In the second case, taxes are not adjusted therefore debt should be seized and its market price adjusts along the path. Even in this case, the deflation cannot be ruled out as an equilibrium. In the third case, it is tacit that at some point in time the ECB is going to buy the countries' debt and proportionally issues units of account in the form of reserves. In this case, the deflation is disallowed as an equilibrium if and only if it is understood that along the deflationary path the supply of reserves is let to growth unboundedly in real terms at a rate higher than $1/\beta$.[20] Indeed, by iterating forward the budget constraint of the government (13), I get that

$$\frac{B_{t-1}}{P_t} = \sum_{T=t}^{\infty} \beta^{T-t}\tau_T + \lim_{T\to\infty}\left\{\beta^T \frac{B_T}{P_T(1+i_T)}\right\}$$

which implies, by using the inequality (15), that

$$\lim_{T\to\infty}\left\{\beta^T \frac{B_T}{P_T(1+i_T)}\right\} > 0.$$

[20] This is also the case if, to start with, B_t denotes central bank's reserves.

The key distinction between the third case and the first two is that the central bank is the only agent in the economy which is not subject to a solvency condition since its liabilities are free of risk in the 'unit of account' regardless of the path of the price level and of central bank's resources.[21] Deflations are ruled out not on the basis of a single action of the fiscal authority but on the coordination between fiscal policy, through the reduction of primary surpluses, and monetary policy, through the expansion of its liabilities. The success of the combination of policies necessarily relies on the power of central bank liabilities

The above argument does not apply in a symmetric way to the case in which an inflationary solution develops. Now the commitment (11) requires the present-discounted value of the real primary surplus to exceed the level of outstanding real obligations if $P_t > P^*$:

$$\sum_{T=t}^{\infty} \beta^{T-t} \tau_T^* = \frac{B_{t-1}}{P^*} > \frac{B_{t-1}}{P_t}.$$

Whether B denotes sovereign debt or central bank's reserves, it does not really matter since in any case this debt is going to be free of risk at the off-equilibrium price $P_t > P^*$.

To understand the credibility of the anti-inflationary commitment I can pose the following two questions. First, has the treasury enough resources to back debt at a higher real value? Second, has the treasury the willingness to provide such an anchor?

Let me answer the first question. Suppose that there is an upper limit $\bar{d}_t$ on how many real resources the treasury is able to raise at any point in time so that

$$\sum_{T=t}^{\infty} \beta^{T-t} \tau_T \leq \bar{d}_t.$$

If $\bar{d}_t$ is less than B_{t-1}/P^* for any finite level of debt B_{t-1} reached in an inflationary path, the treasury does not have enough resources to disallow

[21] Several works in the literature, among which Obstfeld and Rogoff (1983) and Sims (1994), have shown that deflationary solutions can be defeated by setting a target for the supply of money. However, in these analyses, the central bank sets its policy in terms of the supply of money rather than an interest-rate rule. Woodford (1995, 1999, 2001b., 2003 ch. 2) instead assume an interest-rate policy and shows that deflations can be ruled out by targeting the growth of the overall nominal liabilities of the government (including treasury and central bank). This is in line with my analysis with the caveat that the floor should be necessarily put on the path of central bank's liabilities. See also Buiter (2017).

all the inflationary solutions.[22] This means that a policy rule (2) with $\phi > 0$ is at the same time consistent with a stable price level P^* and inflationary equilibria. If the commitment to (2) is irrevocable, it is not even possible for the fiscal authority to backstop inflation by promising to repay debt at a price P^{**} greater then P^*, unless the central bank changes simultaneously (2) using a target P^{**} rather than P^*. Therefore, for countries that have weak fiscal ability, the credibility of the commitment (11) and, at the same time, of (2) can be questioned. In monetary unions with several fiscal authorities, like the EMU, coordination problems can further weaken the overall fiscal capacity, as Sims (1999) has emphasized.

Consider now that there is no upper limit $\bar{d}_t$ on the resources that the treasury can raise. This is anyway not sufficient to disallow inflationary solutions. At the end, on off-equilibrium paths, fiscal policy could passively accommodate an inflationary spiral and save on taxes following own, here unmodelled, incentives. It could also set primary surpluses to target a higher $P^{**} > P^*$, save on taxes, but conflict with the interest-rate policy of the central bank. In this case, either the treasury or the central bank should give up on their policy.

The only possibility for inflationary solutions to be ruled out is that the treasury internalizes the objective of the central bank – to keep prices at P^*– and, thereby, provides a large enough fiscal adjustment in any possible upward deviation. Without this fiscal anchor, the central bank is helpless to counteract inflationary spirals. Though, as shown before, it plays an important role in eliminating deflationary spirals.[23] This is indeed the main message of the fiscal theory of the price level, that a fiscal stimulus to avoid deflations or a fiscal anchor to rule out inflations, are needed to control the price level.

I now turn to describe my proposal whose main contribution is to show that fiscal policy 'activism' is not necessary to control the price level. In the simple model of the next section, actually, treasury debt is always going to be zero. The central bank alone can control the price level disallowing divergent solutions. The key intuition for why this is possible is that every

[22] Note that the level of nominal debt B_{t-1} reached in an inflationary path is higher than that under the constant price P^*, for the same path of real primary surpluses followed until that point in time.

[23] The only case in which the central bank has no role in the fiscal theory of the price level is that in which treasury's liabilities define the 'unit of account'. But, in this case, there is no need of a central bank at all.

monetary policy action has fiscal consequences, implying transfers to the private sector. Whereas the central bank has the special power to increase reserves at its will, without being subject to a solvency condition, solvency, instead, together with the composition of its balance sheet and remittances policy matters for the value of the 'unit of account'. In what follows, however, I am not questioning the key insight of the fiscal theory of the price level, i.e. that either (7) or (8) or (10) are equivalent equilibrium conditions, one of which is needed to determine prices.[24]

4 Central bank theory of the price level

I enrich the model along few dimensions. First, I separate the budget constraints of treasury and central bank. Second, I model explicitly the supply and demand of central bank's reserves. As in the simple model of previous section, money can be set to zero without losing generality. Reserves X_t are held by the consumers. Their budget constraint modifies to

$$\frac{B_t + X_t}{1 + i_t} = B_{t-1} + X_{t-1} + P_t(y - c_t) - P_t\tau_t, \tag{16}$$

with $X_{t_0-1} = 0$. Accordingly, their optimizing behavior now implies the following transversality condition

$$\lim_{t\to\infty} \left\{ R_{t_0,t} \frac{B_t + X_t}{P_t(1 + i_t)} \right\} = 0, \tag{17}$$

which replaces (8).

In the split, the budget constraint of the treasury is given by:

$$\frac{B_t^F}{1 + i_t} = B_{t-1}^F - P_t\tau_t - T_t^C$$

where B_t^F is now treasury's debt with initial condition $B_{t_0-1}^F = 0$ and T_t^C are the nominal remittances received from the central bank, when positive, or transfers made to the central bank, when negative. Central bank's flow budget constraint is instead

$$\frac{B_t^C - X_t^C}{1 + i_t} = B_{t-1}^C - X_{t-1}^C - T_t^C, \tag{18}$$

[24] I am indeed relying on similar wealth effects as those emphasized by Barro (1974).

where B_t^C are central bank's holdings of short-term riskless assets while X_t^C is the supply of central bank's reserves, with initial conditions $B_{t_0-1}^C = X_{t-1}^C = 0$.

Equilibrium in asset markets requires that

$$B_t + B_t^C = B_t^F$$

and

$$X_t = X_t^C.$$

Needless to say, the market of reserves clears in a separate way from that of short-term bonds although both securities face same interest rate in equilibrium. I need now to specify in more details the monetary/fiscal policy regime, given the new elements introduced: this involves to set, in turn, the tax policy τ_t, the remittances policy T_t^C and the central bank's holdings of short-term bonds, B_t^C or the amount of reserves X_t^C issued.

To clarify my proposal at most, I will assume that the treasury is not issuing any debt in contrast with what needed by the fiscal theory of the price level. Given that $B_t^F = 0$ at each date t, taxes completely offset the remittances coming from the central bank

$$\tau_t = -\frac{T_t^C}{P_t}. \tag{19}$$

In fact, the above equation requires that real primary surplus of the treasury is set to zero at each point in time.[25] According to the terminology used by Leeper (1991), the treasury is following a 'passive' fiscal policy, meaning that its own liabilities are appropriately bounded -here always zero– regardless of the value taken by other endogenous variables among which prices. The treasury acts nothing more than an intermediary between the central bank and the private sector, by construction, since it is the first recipient of the remittances of the central bank. Were the central bank having a direct link with the private sector, the intermediary role of the treasury would disappear. In any case, and in a more general model with treasury's debt, the treasury is not meant to be different from any other private agent in the economy issuing debt denominated in the 'unit of account'. It is subject to a solvency condition and has to find enough resources to pay its obligations, in the case debt is risk free, or seize them, otherwise.[26]

[25] According to public accounts, the treasury's primary surplus includes also the remittances received from the central bank.

[26] With a positive supply of treasury debt, the results of the paper are unchanged by

4.1 Ruling out liquidity traps

In this richer model, I will now show how it is possible to rule out deflationary spiral and liquidity traps. There are three main ingredients of the proposal on top of the assumption that the central bank sets the interest rate on reserves. First, the central bank receives an initial capitalization from the private sector, through the treasury. Second, it holds only risk-free bonds in its portfolio while issuing reserves. Third, it commits to transfer all profits to the private sector, again through the treasury, without being subject to any further interference from third parties that ask for higher remittances or attempt to expropriate central bank's net worth. This is the one side of the concept of financial independence I have underlined in the introduction.

I will now describe each element of the proposal in details, show the result and then later discuss the consequences of relaxing each assumption in turn.

First element: the central bank starts at time t_0 with an initial injection of real capital $n_{t_0}^C > 0$ which is collected by the treasury through lump-sump real taxes levied on the consumers, $\tau_{t_0} = n_{t_0}^C$. Therefore the time t_0 remittance is negative and given by

$$\frac{T_{t_0}^C}{P_{t_0}} = -\tau_{t_0} = -n_{t_0}^C. \tag{20}$$

The second element: the central bank issues interest-bearing liabilities, reserves, and holds short-term securities. At time t_0, given the initial injection of net worth, the central bank's balance sheet is:

$$\frac{B_{t_0}^C - X_{t_0}}{(1 + i_{t_0})} = P_{t_0} n_{t_0}^C. \tag{21}$$

Since the liabilities of the central bank are the 'unit of account', as already emphasized, this gives the power to the central bank to set both the interest-rate on reserves and the amount of reserves. The latter can be accomplished by directly specifying the sequence $\{X_t\}_{t=t_0}^{\infty}$ or instead the amount of open-market purchases $\left\{B_t^C\right\}_{t=t_0}^{\infty}$.

Third element: the central bank commits to rebate all profits to the treasury $T_t^C = \Psi_t^C$ at each date $t > t_0$. It follows that time-t profits are given

assuming a 'passive' fiscal policy according to the definition of Benigno and Nisticò (2015) and in line with Leeper (1991).

by

$$\Psi_t^C = \frac{i_{t-1}}{1+i_{t-1}}(B_{t-1}^C - X_{t-1}^C). \tag{22}$$

A first implication of the three assumptions is that central bank's nominal net worth is constant and positive at any time. Define, nominal net worth, N_t^C, at a generic time t as

$$N_t^C \equiv \frac{B_t^C - X_t^C}{(1+i_t)}.$$

Using the latter definition and equation (22) into (18), I obtain that

$$N_t^C = N_{t-1}^C + \Psi_t^C - T_t^C. \tag{23}$$

Applying the remittances rule $T_t^C = \Psi_t^C$, I derive the time invariance of nominal net worth, $N_t^C = N_{t-1}^C = ... = P_{t_0} n_{t_0}^C > 0$.

A second implication is that profits are non-negative, indeed

$$\begin{aligned} \Psi_t^C &= \frac{i_{t-1}}{1+i_{t-1}}(B_{t-1}^C - X_{t-1}^C) \\ &= i_{t-1}N_{t-1} = i_{t-1}P_{t_0}n_{t_0}^C \geq 0, \end{aligned}$$

where I have used the definition of net worth and its time invariance.

This second implication completes the other side of the concept of financial independence. The central bank does not receive any further transfer from the treasury after time t_0 and can actually rebate positive profits when the nominal interest rate is positive.

I now discuss how the three ingredients described above can rule out deflationary spirals. To this end, I study the implications of the above remittances' policy in terms of the path of real taxes faced by the consumer and study the resulting equilibrium prices through the equilibrium condition (10). At time t_0, $\tau_{t_0}^* = n_{t_0}^C$ but then in the following periods transfers from the central bank to the private sector are given by

$$\tau_t^* = -\frac{T_t^C}{P_t} = \frac{\Psi_t^C}{P_t} = -\frac{i_{t-1}}{P_t}P_{t_0}n_{t_0}^C \tag{24}$$

for each $t > t_0$.

Let's see first whether prices $P_{t_0} \geq P^*$ can be equilibria. Given the policy rule (2), nominal interest rates are always positive. Computing the present-discounted value of the remittances (24), it follows that this value is identically equal to the initial capitalization, therefore:

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \tau_t^* = 0.$$

Considering the equilibrium condition (10), I can conclude that any P_{t_0} with $P_{t_0} \geq P^*$ is an equilibrium price.

Let's study instead the case in which $P_{t_0} \leq \beta^{\frac{1}{\phi}} P^*$ and therefore an economy that starts already in a liquidity trap: i.e. $i_t = 0$ at each $t \geq t_0$. Equation (24) shows that central bank's profits are zero as well as the remittances to the treasury, therefore $\tau_t^* = 0$ for each $t > t_0$ while $\tau_{t_0}^* = n_{t_0}^C$. In this case the equilibrium condition (10) is violated. Alternatively, use (9) evaluated at the equilibrium discount factor $R_{t_0,t} = \beta^{t-t_0}$ and plug in $\tau_{t_0}^* = n_{t_0}^C$ and $\tau_t^* = 0$ for each $t > t_0$ to get

$$c_{t_0} = y - (1-\beta) n_{t_0}^C < y. \tag{25}$$

Demand of goods at time t_0 is below supply and therefore prices with $P_{t_0} \leq \beta^{\frac{1}{\phi}} P^*$ are not clearing the market. Similar reasoning can also apply to any other deflationary path in which nominal interest rates start positive and then fall to zero, i.e. for P_{t_0} in the range $\beta^{\frac{1}{\phi}} P^* < P_{t_0} < P^*$.

Another way to see why these solutions are completely ruled out as equilibrium path is to note that under the assumed remittances' rule time invariance of nominal net worth implies that real net worth grows unboundedly, in the case of a deflation, at a rate which is the inverse of β. Indeed,

$$\begin{aligned} \lim_{t\to\infty} \left\{ \beta^{t-t_0} \frac{P_{t_0}}{P_t} N_t^C \right\} &= P_{t_0} n_{t_0}^C \lim_{t\to\infty} \left\{ \beta^{t-t_0} \frac{P_{t_0}}{P_t} \right\} \\ &= P_{t_0} n_{t_0}^C > 0 \end{aligned} \tag{26}$$

where in the first equality I have used the result that nominal net worth is constant and in the second the fact that when $P_{t_0} < P^*$ the rate of deflation is β after some finite period of time. The mirror image of the rise in central bank's net worth is the explosion of the net debt contracted by the consumers

$$\begin{aligned} \lim_{t\to\infty} \left\{ \beta^{t-t_0} \frac{P_{t_0}}{P_t} N_t^C \right\} &= \lim_{t\to\infty} \left\{ \beta^{t-t_0} \frac{P_{t_0}}{P_t} \frac{B_t^C - X_t^C}{(1+i_t)} \right\} \\ &= -\lim_{t\to\infty} \left\{ \beta^{t-t_0} \frac{P_{t_0}}{P_t} \frac{B_t + X_t}{(1+i_t)} \right\}, \end{aligned}$$

where in the first equality I have used the definition of net worth and in the second line equilibrium in the asset markets. Therefore the result in (26) implies violation of the transversality condition (17). This means that the consumers are going to borrow at some point in time more than what they can afford to pay with certainty. Indeed the appropriate natural borrowing limit, that applies in this more general model, requires that the real value of net debt should be at least backed by the present discounted value of net real income, namely

$$\frac{B_t + X_t}{P_{t+1}} \geq -\sum_{T=t}^{\infty} R_{t,T}(y - \tau_{t+1}).$$

Given that τ_t is zero for each $t > t_0$ and since the negative of $(B_t + X_t)/P_{t+1}$ is growing at a rate higher than $1/\beta$, the real value of debt violates the borrowing-limit condition at some point in time.

Key to eliminate deflationary solutions is that the central bank, once capitalized, is committed to rebate all nominal profits to the treasury and keep nominal net worth constant, given the rule $T_t^C = \Psi_t^C$.

I will now put my proposal under the same scrutiny as I did with the fiscal theory of the price level and ask whether these commitments are credible and can be pursued if the deflationary path emerges.

The result that consumption at time t_0 falls below output or, specularly, the need of consumers to borrow in excess of their future resources rest on the fact that the central bank is retaining a positive value of resources in its balance sheet. The critical question to ask is how credible this retention is, given that the missing resources are exactly what prevents the private sector from repaying the amount of debt needed for the deflationary solution to develop as an equilibrium. The treasury could tax the central bank and expropriate entirely its net worth to rebate it to the private sector. This extraordinary measure can be even more justified by noting that during the liquidity trap profits are zero and therefore remittances to the treasury are zero. The treasury could question why the central bank is allowing its net worth to increase in real terms without rebating any resource at all.

The main argument against this observation is that the central bank of this proposal is 'operationally independent', borrowing the terminology and definition of Buiter (2009), i.e. "the freedom or ability of a central bank to pursue its objectives (regardless of who sets them) as it sees fit, without interference or pressure from third parties". The financial independence that

I have outlined in this section is part of the broader concept of operational independence, and it is a two-sided symmetric concept which requires, on the one side, the treasury not to deplete the financial resources of the central bank by taxing it or asking for extraordinary dividends and, on the other side, the central bank not to rely on further external support beyond the initial capitalization. And indeed, if deflations are costly, society has all the incentives to delegate monetary policy to an 'operationally independent' central bank. By doing so, deflations are not even a possibility.

Going more into the details of the model, the deflationary equilibrium fails to arise simply because it is already at time t_0 that the goods market does not clear, as shown by (25). Alternatively, for goods market to clear at time t_0 the private sector needs to borrow more than what it can afford to pay. But this again cannot be an equilibrium under a well-defined consumption problem that forbids Ponzi schemes. Therefore, it is already at time t_0 that counterparties in the credit market have to be sure about the solvency of the private sector and therefore be sure of what happens to the net worth of the central bank – whether at the end is entirely expropriated and rebated to the private sector. If there is even a small probability that this does not happen or even a small amount of capital remains at the central bank then the equilibrium will not form. Therefore the degree of operational independence at which the central bank starts its mandate is critical to make deflationary equilibria unfeasible.

The mechanism underlined in my proposal is different with respect to that of the fiscal theory of the price level. First, note that any proposal of ruling out off-equilibrium path can be read as a violation of the transversality condition. However, the violation implied by my proposal is on the opposite side of (17) with respect to that implied by the fiscal theory of the price level. In the latter, the whole government makes sure that its overall real liabilities explode in real terms if a deflation occurs. This corresponds to an unbounded growth of the real assets of the consumers. The optimization problem of the consumer is well defined but the exhaustion of its intertemporal budget constraint –which is an optimality condition– is violated. In my case, instead, the central bank is letting its real net worth to grow unboundedly which corresponds to an explosion of the debt of the consumers. What is violated is not a first-order condition of the consumer. It is already the optimization problem of the consumer which is not well defined because the no-Ponzi condition is violated.

The three ingredients of the monetary/fiscal policy regime outlined above are key to get rid of the deflationary solutions. First, note that if the central bank does not receive capital at time t_0 – while I maintain the other elements of the monetary-fiscal policy regime unchanged – then deflationary solutions are not excluded as equilibria, as shown in (26) since $N_T = N_{T-1} = ... = 0$. I have already underlined the critical role of the central bank's ability to stick to its remittances' policy without being subject to interference from third parties. Finally, a risk-free composition of the central bank's assets is also key to preserve financial independence and implicitly avoiding any further interferences from the treasury. In Section 5, I will show that by engaging in purchases of risky assets the central bank puts at risk the control of the price level.

4.2 Ruling out inflationary spirals

I now rule out inflationary spirals with a simple amendment to the remittances' policy. I still assume that the central bank receives an initial capital $n_{t_0}^C > 0$ and moreover that the central banks rebates directly all its profits to the treasury following the rule $T_t^C = \Psi_t^C$, but only up to time $\tilde{t}$. As shown before, this implies that

$$\frac{T_t^C}{P_t} = i_{t-1}\frac{N_{t-1}}{P_t} = i_{t-1}\frac{P_{t_0}}{P_t}n_{t_0}^C,$$

for each $t_0 < t < \tilde{t}$. The further assumption is that the central bank commits to switch to the following constant real remittances' policy after and including time $\tilde{t}$

$$\frac{T_t^C}{P_t} = \frac{1-\beta}{\beta}\frac{P_{t_0}}{P^*}n_{t_0}^C. \tag{27}$$

Note that time $\tilde{t}$ can be set far in the future.

Consider now the transversality condition (17) evaluated at the equilibrium discount factor $R_{t_0,t} = \beta^{t-t_0}$ and equilibrium in the asset markets, i.e. $X_t^C = X_t$ and $B_t = -B_t^C$ given that I am still assuming $B_t^F = 0$. Therefore the transversality condition (17) implies that, in equilibrium, central bank's real net worth should be appropriately bounded

$$\lim_{t\to\infty}\left\{\beta^{t-t_0}\frac{B_t^C - X_t}{P_t(1+i_t)}\right\} = \lim_{t\to\infty}\left\{\beta^{t-t_0}\frac{N_t}{P_t}\right\} = 0, \tag{28}$$

where I have used the definition of central bank's net worth.

Exploiting the above condition and the central bank's flow budget constraint (18), I can get that

$$\frac{N_t^C}{P_t} = \sum_{T=t+1}^{\infty} \beta^{T-t} \frac{T_T^C}{P_T}. \tag{29}$$

for each $t \geq t_0$ which is therefore an equilibrium condition and nothing more than the mirror image of (7), given the assumption of 'passive' fiscal policy.

As in the fiscal theory of the price level, the equilibrium condition (29) is a valuation equation that can be used to determine the price level, but now what matters is not treasury's debt nor its primary surplus, but the level of nominal net worth of the central bank and its remittances' policy. However, the most important difference is that N_t^C, being a liability of the central bank, also defines the 'unit of account'.

To see how the proposed remittances' rule determines uniquely the price level at P^*, ruling out inflationary solutions, consider the equilibrium condition (29) at time $\tilde{t}-1$. Since $T_t^C = \Psi_t^C$ for $t_0 < t \leq \tilde{t}-1$, the law of motion of net worth (23) implies that $N_{\tilde{t}-1}^C = P_{t_0} n_{t_0}^C > 0$ and therefore in (29) that

$$\frac{P_{t_0} n_{t_0}^C}{P_{\tilde{t}-1}} = \sum_{T=\tilde{t}}^{\infty} \beta^{T+1-\tilde{t}} \frac{T_T^C}{P_T}.$$

Now, substitute into the right-hand side of the above equation the path of real remittances (27) for each $t \geq \tilde{t}$ to obtain

$$\frac{P_{t_0}}{P_{\tilde{t}-1}} n_{t_0}^C = \frac{P_{t_0}}{P^*} n_{t_0}^C.$$

The above equation determines $P_{\tilde{t}-1} = P^*$ if and only if $n_{t_0}^C \neq 0$. The difference equation (3) therefore implies that the only equilibrium is one in which prices are forever at the target P^*. Inflationary spirals are ruled out by the simple threat that the central bank is committed at some point in time to back the real value of its net worth at the desired level P^*. The initial capitalization of the central bank, $n_{t_0}^C > 0$, is again important to get the result as the above equation shows. Moreover, the remittances rule $T_t^C = \Psi_t^C$ and the assumption that the central bank invests only in risk-free assets are also necessary conditions, since they both imply that the central bank can keep the value of nominal net worth positive over time and therefore being able to generate a positive stream of remittances in the future.

One way to implement the above proposal is to allow the central bank to sell, at time $\tilde{t}-1$, shares of its capital to the private sector and entitle each owner of one dollar unit of its capital to receive a constant stream of real dividends each equal to $(1-\beta)/(\beta P^*)$. Given that output is constant across time and the financial market real rate is just $1/\beta$, the market value a constant stream of real dividends $(1-\beta)/(\beta P^*)$ is $1/P^*$. If prices at time $\tilde{t}-1$ are above P^*, there can be arbitrage opportunities. Consumers can borrow in the financial markets $1/P^*$ unit of goods at $\tilde{t}-1$ and promise to pay a constant real stream $(1-\beta)/(\beta P^*)$. The can sell the goods for $P_{\tilde{t}-1}/P^* > 1$ dollars. They can invest one dollar in the central bank to receive a stream of dividends that exactly offset the payment to make while they remain with $P_{\tilde{t}-1}/P^* - 1$ dollars that can be used to buy goods at time $\tilde{t}-1$. Arbitrage opportunities are eliminated only when $P_{\tilde{t}-1} = P^*$.

First note that the described implementation of the proposal shares similarities with Hall and Reis (2016)'s idea that the central bank can determine the price level by fixing the real payment on each dollar unit of reserves. Indeed, even in their case, arbitrage opportunities arise if the price level does not equalize the real return on reserves to the market real rate. However, Hall and Reis (2016) recommend that the central bank should always use a real-payment policy on reserves in place of a nominal interest-rate rule while, instead, I maintain the more conventional nominal interest-rate policy and use the real dividend policy only as a threat to eliminate inflationary solutions.

I now turn to discuss the credibility of central bank's commitment on off-equilibrium paths. The central bank is committed from time $\tilde{t}$ onwards to transfer real resources by an amount that exceeds the real value of its net worth at current prices if an inflationary path ($P_{\tilde{t}-1} > P^*$) develops:

$$\sum_{T=\tilde{t}}^{\infty} \beta^{T-\tilde{t}+1} \frac{T_T^C}{P_T} = \frac{N_{\tilde{t}-1}^C}{P^*} > \frac{N_{\tilde{t}-1}^C}{P_{\tilde{t}-1}}.$$

The challenge of this type of commitment is at a first sight similar to that faced by the fiscal theory of the price level when trimming deflationary paths but there are two important differences. First, in this case, there is no need of any coordination between the monetary and fiscal authority, it is just the central bank that needs to fulfill the commitment. Second, as a consequence, the central bank can directly rely on the power of issuing its liabilities at will, as I have already emphasized.

The credibility of the central bank's commitment can be understood by answering two questions. First, can the central bank at time $\tilde{t}$ generate a stream of real resources equal to (27) forever? The answer is positive since it can issue an increasing amount of its reserves – which are risk-free – in a way that their real value grows unboundedly at a rate equal or higher than $1/\beta$. In real term, this is exactly the type of commitment that is needed under the fiscal theory of the price level to combat a deflation in order to support the fiscal expansion. But, here, there is no need of coordination between the monetary and fiscal authority.[27]

The second question is the following: suppose that at time $\tilde{t}-1$ the price level is $P_{\tilde{t}-1} > P^*$, is it really credible to expect that the central bank follows its threat (27) forever or instead will backstop prices at $P_{\tilde{t}-1}$? This is a question similar to that asked when I was evaluating the treasury's commitment to rule out inflationary solutions under the fiscal theory of the price level. The main difference is that in this case there is no possible conflict between the central bank and the treasury since all is about the willingness of the central bank to fulfill its price mandate P^*. The strength of this willingness is directly related to that of the commitment to follow an active interest rate rule (2), i.e. with $\phi > 0$. Indeed, if the central bank sets the interest rate rule as in (2) with $\phi > 0$ and an inflationary path develops, to backstop the price level at $P_{\tilde{t}-1}$ the central bank has necessarily to change the policy rule (2).[28] Therefore, the two commitments – to always follow the policy rule (2) with $\phi > 0$ and to switch to the remittances' rule (27) after time $\tilde{t}-1$ – imply that the price level to expect at time $\tilde{t}-1$ is either P^* or infinity.[29] But, why should the private sector expect a barter economy – a

[27] The other difference is that in this case reserves have to increase in nominal terms more than the rate at which prices are growing.

[28] The case of an interest-rate peg, $\phi = 0$, presents an interesting difference since the interest-rate policy does not need to change if the central bank wants to stabilize prices at $P_{\tilde{t}-1} > P^*$. However, the credibility of the price target P^* can be easily questioned in this case.

[29] To see that the price is going to be infinity consider the implications of the arbitrage argument outlined above when $P_{\tilde{t}-1} > P^*$. There are indeed excess resources to spend which push up the price level. Therefore the price level is already infinite at time t_0. In the literature, there are other examples that rule out inflationary solutions by the threat that no finite inflation rate can arise at a finite point in time. In Obstfeld and Rogoff (1983), this is obtained by assuming that the demand of real money balances remains bounded below in an inflationary spiral, which is somewhat implausible. Woodford (2003) instead assumes that the functional form of the interest-rate policy becomes unboundedly

zero value of the 'unit of account' – after having capitalized the central bank with real resources $n^C_{t_0}$? It will completely waste these resources.[30] Therefore the initial real capitalization of the central bank, the commitment to an active interest rate rule, together with all the other elements discussed above, can anchor the price level to P^*.

I present now a variation of the above discussion where I strengthen the credibility of the commitment using additional instruments. Starting from the same time $\tilde{t}$, the central bank could impose a reserve requirement on the debt issued by the private sector, while maintaining all the other features specified above. This is possible since the private sector, as of time $\tilde{t}-1$, is net debtor with respect to the central bank, indeed $N^C_{\tilde{t}-1} > 0$ implies that $B_{\tilde{t}-1}+X_{\tilde{t}-1} < 0$. Denote with X^r_t the required reserves which are remunerated at i^r_t, below the market rate. After time $\tilde{t}$, central bank's net worth is now:

$$N^C_t = \frac{B^C_t - X^C_t}{(1+i_t)} - \frac{X^r_t}{(1+i^r_t)}.$$

Taking into account the transversality condition (28), I can replace the equilibrium condition (29), at time $\tilde{t}-1$, with

$$\frac{N^C_{\tilde{t}-1}}{P_{\tilde{t}-1}} + \sum_{T=\tilde{t}}^{\infty} \beta^{T-\tilde{t}+1} \left(\frac{i_T - i^r_T}{1+i_T}\right) \frac{X^r_T}{P_T} = \sum_{T=\tilde{t}}^{\infty} \beta^{T-\tilde{t}+1} \frac{T^C_T}{P_T}. \tag{30}$$

The central bank has now two additional intertwined instruments, X^r_t and i^r_t that can be set for each period t following $\tilde{t}$. One possibility is to assume that they are implicitly defined by the following condition

$$\sum_{T=\tilde{t}}^{\infty} \beta^{T-\tilde{t}+1} \left(\frac{i_T - i^r_T}{1+i_T}\right) \frac{X^r_T}{P_T} = (1+\epsilon) \left(\frac{N^C_{\tilde{t}-1}}{P^*} - \frac{N^C_{\tilde{t}-1}}{P_{\tilde{t}-1}}\right) \tag{31}$$

for any finite $P_{\tilde{t}-1}$ and for some positive ϵ, which can be considered small enough. Substituting this policy into (30), I obtain

$$\frac{N^C_{\tilde{t}-1}}{P^*} + \epsilon \left(\frac{N^C_{\tilde{t}-1}}{P^*} - \frac{N^C_{\tilde{t}-1}}{P_{\tilde{t}-1}}\right) = \sum_{T=\tilde{t}}^{\infty} \beta^{T-\tilde{t}+1} \frac{T^C_T}{P_T} > \frac{N^C_{\tilde{t}-1}}{P^*}$$

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large at a finite level of P_t/P^*. In my analysis, instead, the central bank still maintains the commitment to (2) which implies a bounded interest rate given a bounded P_t/P^*.

[30] This is the first instance in the analysis in which it is relevant to assume that the initial capitalization of the central bank is real rather than nominal. It is also a waste of resources to pay $n^C_{t_0}$ and then doubt the fact that the central bank can increase reserves unboundedly because that will lead to currency substitution.

which shows that the central bank has now more resources than what it needs to fulfill the commitment (27) if an inflationary spiral develops. It is again the case that by following the threat (27) after time $\tilde{t}-1$, the central bank can determine uniquely the price level at P^* forever. To get the result, substitute (27) into the above equation. The important difference with respect to the previous solution is that now the central bank is committed to rebate less resources to the private sector than what it has in its balance sheet. Moreover given the two commitments – to an active interest-rate policy (2) (with $\phi > 0$) and to the transfer rule (27) for each $t \geq \tilde{t}$ – the only price to expect at time $\tilde{t}-1$ is P^*. The mechanism that rules out inflationary solutions acts now through a contraction of aggregate demand, because of the resources withheld by the central bank, until the target price level P^* is reached.

To understand condition (31) in a simple example, define the real value of required reserves as $x_t^r \equiv X_t^r/P_t$ and the spread between the nominal interest rate on excess and required reserves $k_t \equiv (1+i_t)/(1+i_t^r) > 1$. Set them constant over time to obtain that

$$\left(1-\frac{1}{k}\right)x^r = (1+\epsilon)\frac{1-\beta}{\beta}\left(\frac{N_{\tilde{t}-1}^C}{P^*} - \frac{N_{\tilde{t}-1}^C}{P_{\tilde{t}-1}}\right).$$

Given that $N_{\tilde{t}-1}^C = P_{t_0}n_{t_0}^C > 0$, the above equation determines the combination of x^r and k to use conditional on the level that prices $P_{\tilde{t}-1}$ would reach off equilibrium. Since the remittances rule and the balance-sheet policy followed until time $\tilde{t}-1$ imply that $N_{\tilde{t}-1}^C > 0$, it is indeed possible to find $x^R > 0$ and $k > 1$.

To evaluate the credibility of these additional instruments, it is key to understand what entitles the central bank of the power to tax the financial sector. The answer is again in the special characteristics of its liabilities that define the 'unit of account' and are risk-free by definition. The financial sector can also manufacture risk-free securities but can be subject to run due to the possible illiquidity of the resources used to back them. The central bank is the only institution that can credibly be the lender of last resort in the 'unit of account' and can therefore solve illiquidity problems. By this virtue, it can exert a taxation power on the financial sector.

Considering also the results of the previous, I have therefore found a specification of the monetary/fiscal policy regime which ensures uniqueness of equilibrium and in which the central bank can control the price level without any additional support from the treasury beyond the initial capitalization.

5 Unconventional open-market operations and control of the price level

I extend the model in two directions. First, I assume that money provides liquidity services to the consumer which are modelled as direct utility derived from real money balances. Second, I add long-term securities and allow the central bank to hold them. Details of the model and equilibrium conditions are left to the Appendix. I here outline the main changes before studying the implications for price determination.

The household's budget constraint modifies to:

$$M_t + \frac{B_t + X_t}{1 + i_t} + Q_t D_t \leq M_{t-1} + B_{t-1} + X_{t-1} + (1 - \varkappa_t)(1 + \delta Q_t) D_{t-1} + P_t(y - c_t) - T_t^F. \tag{32}$$

where D_t indicates long-term securities issued at a price Q_t. The security available has decaying coupons: by lending Q_t units of currency at time t, geometrically decaying coupons are delivered equal to $1, \delta,\ \delta^2, \delta^3...$ in the following periods and in the case of no default.[31] The variable $\varkappa_t$ on the right-hand side of (32) captures the possibility that long-term securities can be partially seized by exogenous default.

Since consumers get utility from real money balances, the following demand schedule of real money balances can be obtained from their first-order conditions

$$\frac{M_t}{P_t} \geq L\left(c_t, i_t\right),$$

which holds with equality whenever $i_t > 0$. The function $L(\cdot, \cdot)$ is defined in the Appendix and is non-decreasing in c and non-increasing in i with $L\left(c_t, 0\right) = \bar{m}$. Absence of arbitrage opportunities implies that the price Q_t of long-term bonds satisfies

$$Q_t = \beta \frac{U_c(c_{t+1})}{U_c(c_t)} \frac{P_t}{P_{t+1}} (1 - \varkappa_{t+1})(1 + \delta Q_{t+1})$$

from which a "fundamental" solution follows by forward iteration:

$$Q_t = \sum_{T=t}^{\infty} \delta^{T-t} \beta^{T+1-t} \left(\frac{P_t}{P_{T+1}}\right) \prod_{j=t+1}^{T+1} (1 - \varkappa_j), \tag{33}$$

[31] The stock of long-term asset follows the law of motion $D_t = Z_t + (1 - \delta) D_{t-1}$, where Z_t is the amount of new long-term lending, if positive, supplied at time t. See among others Woodford (2001b).

where I have also used equilibrium in the goods market. In a perfect-foresight equilibrium the return on long-term bonds is also equal to the short-term interest rate, i.e. $r_{t+1} = i_t$ with the return on long-term bonds defined by $r_{t+1} \equiv (1 - \varkappa_{t+1})(1 + \delta Q_{t+1})/Q_t - 1$.

I still continue to assume that the treasury is not issuing any debt and follows the simple fiscal rule (19). The central bank can instead invest also in long-term securities, D_t^C. Net worth, N_t^C, and profits, Ψ_t^C, are now given by

$$N_t^C = Q_t D_t^C + \frac{B_t^C}{1+i_t} - M_t^C - \frac{X_t^C}{1+i_t}, \tag{34}$$

$$\Psi_t^C = i_{t-1}(N_{t-1}^C + M_{t-1}^C) + (r_t - i_{t-1}) Q_{t-1} D_{t-1}^C. \tag{35}$$

Profits show an additional component that represents the excess gains or losses of holding long-term securities with respect to a riskless portfolio. Since the excess return on these securities can be negative due to unexpected shocks, the latter component may as well be negative – the more so the larger are the holdings of long-term securities – producing income losses for the central bank.

Combining (23), (34) and (35) the central bank's flow budget constraint follows:

$$Q_t D_t^C + \frac{B_t^C}{1+i_t} - M_t^C - \frac{X_t^C}{1+i_t} = (1-\varkappa_t)(1+\delta Q_t) D_{t-1}^C + B_{t-1}^C - X_{t-1}^C - M_{t-1}^C - T_t^C,$$

given initial conditions $D_{t_0-1}^C, B_{t_0-1}^C, X_{t_0-1}^C, M_{t_0-1}^C$ all equal to zero.

In this more general model, the equilibrium condition (29) is replaced by

$$\frac{N_t^C}{P_t} + \sum_{T=t}^{\infty} \beta^{T-t} \frac{i_T}{1+i_T} \frac{M_T}{P_T} = \sum_{T=t+1}^{\infty} \beta^{T-t} \frac{T_T^C}{P_T}, \tag{36}$$

showing that the value of the central bank is given by the sum of its net worth and the seigniorage revenues. The latter source arises because of the liquidity benefits that real money balances provide to the consumers.

5.1 With treasury's support

By holding long-term bonds, the central bank can be subject to income losses. However, the results of Section 5 can hold even in this more general framework. Key, however, is to interpret the tax rule (19) in a symmetric way. In particular, the rule implies that the treasury is committed to transfer resources to the central bank in the case of negative profits.[32] Given the initial capitalization and the commitment to a remittances' rule of the type $T_t^C = \Psi_t^C$ – which now implies a transfer from the treasury whenever $\Psi_t^C < 0$ – central bank's nominal net worth remains constant and all the discussion of Sections 4.1 and 4.2 to eliminate deflationary and inflationary spirals applies in this more general context. The only amendment is that the real remittances' policy (27) should be replaced by

$$\frac{T_t^C}{P_t} = \frac{1}{\beta}\frac{i_{t-1}}{1+i_{t-1}}\frac{M_{t-1}}{P_{t-1}} + \frac{1-\beta}{\beta}\frac{P_{t_0}}{P^*}n_{t_0}^C, \tag{37}$$

in a consistent way with what should be required by the new equilibrium condition (36).

However, in this case, the central bank is no longer financially independent from the treasury which brings about the risk that the central bank could be asked to remit additional dividends at the treasury's will. What is going to be weakened, in this case, is the credibility of the commitment that rules out deflationary paths, as discussed in Section 4.1. If this weakness is understood by the private sector then deflations can develop unraveling the uniqueness of equilibrium.

The result of this section can be consistent with the story of a central bank that undertakes unconventional open-market operations with a deflation going on. In this environment, it is also possible that the central bank derives profits from its holdings of risky assets, as a consequence of unexpected deflationary shocks, and can therefore rebate income to the treasury. However, it is understood that in the case of losses – following perhaps a future exit from a policy of zero nominal interest rates – the treasury stands ready to support the central bank. This implicit support might be enough to undermine the financial independence of the central bank during the liquidity trap because it suggests that it could be in the treasury's ability to expropriate central bank's net worth. Although these raids are not in the observation period, the expectation that they will occur is sufficient to validate the deflationary path.

[32] An interesting example in the recent financial crisis of explicit treasury's support is that of the Bank of England which in January 2009 established a wholly-owned subsidiary with the responsibility of buying private and public long-term securities. The company is fully indemnified by the Treasury since any financial losses are borne by the Treasury and any gains are owed to the Treasury.

In the next section, I am going to analyze the case in which the central bank retains financial independence by refusing any treasury's support beyond the initial capitalization. I am going to show that by purchasing risky securities it loses anyway control of the price level.

5.2 Without treasury's support

I still maintain the assumption that at time t_0 the treasury provides the initial capital through which the central bank starts its operations. However, after time t_0, remittances are assumed to be non-negative, $T_t^C \geq 0$, excluding any possible support from the treasury. In particular, I assume that the central bank transfers all its income to the treasury provided nominal net worth is not below the initial level $\bar{N}$.[33] But, as nominal net worth falls below $\bar{N}$ because of negative profits, the central bank rebuilds it by retaining earnings up to the point in which the initial level $\bar{N}$ is recovered. Therefore for each $t > t_0$ $T_t^C = \max(\Psi_t^C, 0)$ whenever $N_t^C \geq \bar{N}$ and $T_t^C = 0$ if $N_t^C < \bar{N}$. This remittances' policy has a real-world counterpart in the deferred-asset regime currently used by the Federal Reserve System for which, whenever capital falls, the central bank stops making remittances and accounts for a deferred asset in its balance sheet paid later by retained earnings. Only once the deferred asset is paid in full, the central bank returns to rebate profits to the treasury.

5.2.1 Interest-rate risk

First, I analyze the case in which the central bank faces losses on its balance-sheet because of unforeseen movements in the price of long-term assets triggered by an unexpected change in the price level which can then be self-fulfilling. I am going to show that uniqueness is no longer guaranteed since inflationary equilibria exist whereas deflationary solutions can still be ruled out.

Let me start first by eliminating deflationary paths. Assume that the private sector expects that $P_t < P^*$ at a time t and therefore a deflationary path implied by the Taylor's rule combined with the Fisher's equation. Given this switch in expectations, the price of long-term bonds rises and the central bank benefits of a capital gain on the holdings of long-term securities. Under

[33] I define $\bar{N} \equiv P_{t_0} n_{t_0}^C$.

the assumed remittances' policy the capital gain is immediately rebated to the treasury. Net worth remains constant and (26) ensures that solutions with deflationary paths are ruled out.

What will instead happen if the private sector expects $P_t > P^*$ at time t and thereafter an increasing path of prices consistent with the policy rule (2) and the Fisher's equation? The time-t price of long-term bonds would unexpectedly fall causing income losses for the central bank. Note that using (3) into (33), the price of long-term bonds Q_t can be expressed as a function of P_t, that is $Q_t = Q(P_t)$ which is decreasing in P_t.[34] I can also write the return on long-term securities and central bank's profits at time t as a function of P_t, that is $r(P_t)$ and $\Psi^C(P_t)$ respectively.

Assume that the fall in the return on long-term bonds is enough to turn profits to be negative, i.e.

$$\Psi^C(P_t) = i_{t-1}(N_{t-1}^C + M_{t-1}^C) + (r(P_t) - i_{t-1})Q_{t-1}D_{t-1}^C < 0.$$

Since there is no treasury's support, the income loss translates into a fall in central bank's net worth. In the previous section, I showed that these inflationary spirals are not equilibria since the central bank is able to back equilibrium prices by using the threat (37) which implies that the central bank has enough resources to promise to pay positive real remittances back to the treasury and then to the consumers. Here these resources might be at risk since central bank's net worth can instead fall. Given its dependence on r_t and therefore on Q_t, the level reached by central bank's net worth at time t is also a function of P_t

$$N^C(P_t) = N_{t-1}^C + \Psi^C(P_t) < N_{t-1}^C, \tag{38}$$

which follows from (23) where I have implicitly assumed that central bank's income is negative and therefore that remittances are zero consistently with what prescribed by the deferred-asset regime.

Define now the seigniorage flow at time t as

$$s(P_t) \equiv \frac{\max[i(P_t), 0]}{1 + \max[i(P_t), 0]} L(i(P_t), y),$$

which is also a function of P_t since the nominal interest is a function of P_t through (2). Let $\mathcal{S}_t$ be the discounted value of seigniorage

$$\mathcal{S}_t \equiv \sum_{T=t}^{\infty} \beta^{T-t} s(P_T).$$

[34] In this subsection, I am assuming that $\varkappa_t = 0$ at all times.

Using (3), I can also write $\mathcal{S}_t$ as function of P_t, i.e. $\mathcal{S}(P_t)$.

Given the above definition, consider now the equilibrium condition (36). If the value of the central bank (the left hand side) is negative, it is not possible to deliver a positive discounted value of remittances to the treasury. In particular the central bank has insufficient backing to defeat an adverse shift in expectations when the following inequality holds

$$\frac{N^C(P_t)}{P^*} + \mathcal{S}(P^*) < 0. \tag{39}$$

In the above expression, nominal net worth is evaluated at the level it would reach if expectations at time t were to shift to an inflationary path as implied by (38). Instead, seigniorage and prices are evaluated at the desired price level, $P_t = P^*$ at all times. If (39) holds, inflationary equilibria are not defeated.[35]

The above reasoning does not exclude that the solution $P_t = P^*$ remains an equilibrium, indeed the value of the central bank at P^* is positive

$$\frac{N^C(P^*)}{P^*} + \mathcal{S}(P^*) = \frac{\bar{N}}{P^*} + \mathcal{S}(P^*) > 0,$$

given that net worth $N^C(P^*)$ is kept constant at the initial value. It follows that the equilibrium condition (36) holds for $P_t = P^*$ and for each t.

Finally note that the results of this section are related to Del Negro and Sims (2015) but with an important difference. In their analysis, multiplicity appears as a shift to a different interest-rate rule, since it is P^* that changes across equilibria. In my analysis, the policy rule remains unchanged and the multiplicity arises along the multiple solutions that (3) implies, given the inability of the central bank alone to trim some of these paths using internal resources.

5.2.2 Credit risk

I consider now the consequences of an unexpected realization of a credit event showing that the stationary solution $P_t = P^*$ stops to be an equilibrium

[35] Moreover, note that (39) with an equality sign defines a threshold $\tilde{P}_t$ such that for all $P_t \geq \tilde{P}_t$, the inequality holds. It is easy to see that the threshold $\tilde{P}_t$ is lower, the higher the holdings of long-term bonds are, the lower the seigniorage revenues are, and the lower the initial level of capital is.

when the credit event is sizeable whereas divergent solutions (inflationary and deflationary) might emerge now as equilibria.

Starting from a perfect foresight equilibrium in which $\varkappa_t = 0$ at all times, assume that at time t long-term securities are unexpectedly seized, even partially, at the rate $0 < \varkappa \leq 1$.[36] The time-t return on long-term bonds unexpectedly falls which could lead to negative profits and to a fall in net worth. Note that if central bank's profits remain positive after the credit event then the analysis of the previous subsection would still apply, with deflationary solutions which are eliminated and inflationary paths that cannot be completely ruled out.

I consider the most interesting case in which $\Psi^C(\varkappa, P^*) < 0$ where now the profit function depends also on $\varkappa$

$$\Psi^C(\varkappa, P_t) = i_{t-1}(N_{t-1}^C + M_{t-1}^C) + (r(\varkappa, P_t) - i_{t-1})Q_{t-1}D_{t-1}^C$$

through the dependence of the return function $r(\varkappa, P_t)$ on $\varkappa$. Central bank's net worth in the case of negative profits is now

$$N^C(\varkappa, P_t) = N_{t-1}^C + \Psi^C(\varkappa, P_t).$$

The first result is that the stationary solution, $P_t = P^*$ forever, is no longer an equilibrium if and only if

$$\frac{N^C(\varkappa, P^*)}{P^*} + \mathcal{S}(P^*) < 0. \tag{40}$$

If the credit event is strong enough, the reduction in net worth can be so substantial that seigniorage revenues are not sufficient to turn the overall value of the central bank positive at the desired level P^*. In particular condition (40) implies that the shock is large enough to bring net worth to a negative value at the desired price level, $N^C(\varkappa, P^*) < 0$. If (40) holds, then the equilibrium condition (36) shows that no equilibrium can form at a constant price level P^* absent treasury's support. What else can happen? Can inflationary solution still be equilibria maintaining the policy rule (2)?

Yes, if the following condition holds for some $P_t > P^*$

$$\frac{N^C(\varkappa, P_t)}{P_t} + \mathcal{S}(P_t) > 0. \tag{41}$$

[36] I could alternatively assume that private agents expect at time t that $\varkappa_T > 0$ at some future date $T > t$.

Note that the above condition can only be valid if the seigniorage revenues that the central bank gets under the inflationary path are sufficiently large, given that net worth is decreasing with P_t, i.e. $N^C(\varkappa, P_t) < N^C(\varkappa, P^*)$ for $P_t > P^*$.[37]

Maintaining the policy rule (2) unchanged, I now investigate whether deflationary solutions can instead develop. To simplify the exposition, I discuss the existence of a permanent liquidity trap that starts at the time in which the credit shock hits.[38] I guess and verify that a necessary and sufficient condition is that $N^C(\varkappa, P_t) = 0$ for some P_t with $P_t < \beta^{1/\phi} P^*$.

If $P_t < \beta^{1/\phi} P^*$ nominal interest rates are at zero from t onwards, given an interest-rate policy (2) with $\phi > 0$, and therefore also seigniorage revenues are zero, $\mathcal{S}(P_t) = 0$ for each future t. Moreover profits (35) after period t are zero and remittances are also zero, since net worth is below $\bar{N}$. Therefore (23) implies that net worth remains constant at zero. This is an equilibrium because (36) is satisfied in each period by zero net worth, zero seigniorage revenues and zero remittances. The necessity of the condition can be seen by noting that (26) is violated once evaluated at the deflation rate β if it assumed by contradiction that $N^C(\varkappa, P_t) \neq 0$ for all $P_t < \beta^{1/\phi} P^*$. Indeed, if $N^C(\varkappa, P_t) \neq 0$ at time t and $i_t = 0$ forever then zero profits implies, using (23), that net worth remains different from zero in the long run violating then (26).

To wrap up, if the central bank maintains the policy rule (2) with $\phi > 0$ and the inequality (40) holds, the stationary solution is no longer an equilibrium, inflationary and deflationary equilibria can instead develop. If the central bank wants still to pursue a price stability policy, it has to change its policy rule and can replace the desired level P^* in (2) with $P^{**} < P^*$.[39]

Finally consider the opposite case in which

$$\frac{N^C(\varkappa, P^*)}{P^*} + \mathcal{S}(P^*) > 0.$$

[37] If (41) holds at $P_t > P^*$, the equilibrium condition (36) implies that the presented discounted value of remittances to the treasury is positive notwithstanding the credit event. Under a deferred assets regime and in a perfect-foresight equilibrium, the latter result implies that central bank's net worth returns to its initial level $\bar{N}$ in a finite period of time.

[38] The analysis can be generalized to allow for a transition path toward the liquidity trap.

[39] P^{**} should be below P^* because this is the only way through which the sign of (40) can be overturned considering that the seignorage revenues at different stable prices are the same, i.e. $\mathcal{S}(P^*) = \mathcal{S}(P^{**})$.

The solution with $P_t = P^*$ remains an equilibrium but the multiplicity persists. Inflationary equilibria still exist following the discussion of previous subsection. Deflationary equilibria instead exist if and only if $N^C(\varkappa, P^*) < 0$ and therefore disappear if the credit losses are small enough.

6 Conclusion

I have described a monetary/fiscal policy regime that can uniquely determine prices in a simple endowment monetary economy. The important feature of the regime is that once the central bank is appropriately designed with an initial level of capital, a specified remittances' policy and the requirement of holding only riskless securities, and maintains financial independence from third parties, then it is equipped with all the relevant tools to defeat deflationary and inflationary spirals without the need of fiscal support. The elements underlined are not new compared with the evidence on how central banks are designed and some of them are consistent with what economists have been arguing for hundreds years.[40] What is new is that they can determine uniquely a stable price level, once combined with a Taylor's rule or interest-rate pegs, something that the related literature has hardly ever managed to achieve without treasury 'activism'.

This work confirms the tendency of the last twenty years to establish central banks that are more and more independent from the treasuries and other third-party interferences.

The proposal of this work is based on commitments and threats which I have shown are robust to off-equilibrium paths. However, they might be subject to public debate or renegotiations with the government along the way. I leave for future works these political-economy considerations.

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A Appendix

I present in this Appendix the model of Section 5 discussing in turn the consumers' problem, the treasury and the central bank and then characterizing the equilibrium conditions. The models of Section 2, 3, and 4 follow with the appropriate simplifications.

A.1 Consumers

Consumers have preferences:

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[U(c_t) + V\left(\frac{M_t}{P_t}\right) \right] \tag{A.1}$$

where β is the intertemporal discount factor with $0 < \beta < 1$, c is a consumption good and $U(\cdot)$ is a concave function, twice continuously differentiable, increasing in c; $V(\cdot)$ is a non-decreasing twice-continuously differentiable function of real money balances with $V_m(\cdot) = 0$ for $M_t/P_t \geq \bar{m}$ where $\bar{m}$ indicates a finite level of money balances at which there is satiation.

The consumers' budget constraint is:

$$M_t + \frac{B_t + X_t}{1 + i_t} + Q_t D_t \leq M_{t-1} + B_{t-1} + X_{t-1} + (1 - \varkappa_t)(1 + \delta Q_t) D_{t-1} + P_t(y - c_t) - T_t^F. \tag{A.2}$$

Consumers can invest their financial wealth in money M_t issued by the central bank. They can also invest in interest-bearing reserves, X_t, issued as well by the central bank at the risk-free nominal interest rate i_t and can lend or borrow using short-term securities, B_t, at the same interest rate i_t. D_t indicates holdings of long-term securities issued at a price Q_t. The security available has decaying coupons: by lending Q_t units of currency at time t, geometrically decaying coupons are delivered equal to $1, \delta,\ \delta^2, \delta^3...$ in the following periods and in the case of no default.[41] The variable $\varkappa_t$ on the right-hand side of (A.2) captures the possibility that long-term securities can be partially seized by exogenous default. y is a constant endowment of the only good traded; T_t^F are lump-sum taxes levied by the treasury. There are

[41] The stock of long-term asset follows the law of motion $D_t = Z_t + (1-\delta)D_{t-1}$, where Z_t is the amount of new long-term lending, if positive, supplied at time t. See among others Woodford (2001).

no financial markets before time t_0, therefore B_{t_0-1}, X_{t_0-1}, M_{t_0-1}, D_{t_0-1} are all equal to zero.

The consumers' problem is subject to a borrowing limit of the form

$$\lim_{T\to\infty}\left\{R_{t_0,T}\left(M_T+\frac{B_T+X_T}{1+i_T}+Q_TD_T\right)\right\}\geq 0 \tag{A.3}$$

and to the bound

$$\sum_{T=t_0}^{\infty}R_{t_0,T}\left\{P_Tc_T+\frac{i_T}{1+i_T}M_T\right\}<\infty \tag{A.4}$$

since there is no limit to the ability of households to borrow against future income.

Households choose consumption, and asset allocations to maximize utility (A.1) under constraints (A.2), (A.3), (A.4) given the initial conditions. The set of first-order conditions imply the Euler equation

$$\frac{U_c(c_t)}{P_t}=\beta(1+i_t)\frac{U_c(c_{t+1})}{P_{t+1}} \tag{A.5}$$

at each time $t\geq t_0$ assuming interior solutions and the following demand of real money balances

$$\frac{M_t}{P_t}\geq L\left(c_t,i_t\right)$$

with

$$i_t\geq 0$$

at each time $t\geq t_0$, in which at least one of the two inequalities above must hold with equality at any time. The function $L(\cdot,\cdot)$ is defined by $L(\cdot,\cdot)\equiv V_m^{-1}(U_c(c)i_t/(1+i_t))$ which is non decreasing in c and non-increasing in i with $L\left(c_t,0\right)=\bar{m}$.

Absence of arbitrage opportunities implies that

$$Q_t=\beta\frac{U_c(c_{t+1})}{U_c(c_t)}\frac{P_t}{P_{t+1}}(1-\varkappa_{t+1})(1+\delta Q_{t+1}) \tag{A.6}$$

from which a "fundamental" solution for long-term bond prices follows:

$$Q_t=\sum_{T=t}^{\infty}\delta^{T-t}\beta^{T+1-t}\frac{U_c(c_{T+1})}{U_c(c_t)}\left(\frac{P_t}{P_{T+1}}\right)\prod_{j=t+1}^{T+1}(1-\varkappa_j),$$

at each time $t \geq t_0$.

In a perfect-foresight equilibrium the return on long-term bonds is also equal to the short-term interest rate as shown by combining (A.5) and (A.6)

$$r_{t+1} = i_t \tag{A.7}$$

with the return on long-term bonds defined by $r_{t+1} \equiv (1-\varkappa_{t+1})(1+\delta Q_{t+1})/Q_t - 1$.

To conclude the characterization of the consumer's problem, a transversality condition applies and therefore (A.3) holds with equality, given the equilibrium nominal stochastic discount factor

$$R_{t_0,T} = \beta^{T-t_0} \frac{U_c(c_T)}{U_c(c_{t_0})} \frac{P_{t_0}}{P_T}.$$

A.2 Treasury

The treasury raises lump-sum taxes T_t^F (net of transfers) from the private sector and receives remittances T^C (when T^C is positive) or makes transfers to the central bank (when T^C is negative). The treasury can finance its deficit through short-term debt (B^F) at the price $1/(1+i_t)$, facing the following flow budget constraint

$$\frac{B_t^F}{1+i_t} = B_{t-1}^F - T_t^F - T_t^C$$

given initial condition $B_{t_0-1}^F = 0$. To simplify the analysis, I assume that the treasury does not issue long-term securities.

A.3 Central Bank

The central bank can invest in short and long-term securities, B_t^C and D_t^C, by issuing money and reserves, M_t^C and X_t^C. Net worth, N_t^C is defined as

$$N_t^C \equiv Q_t D_t^C + \frac{B_t^C}{1+i_t} - M_t^C - \frac{X_t^C}{1+i_t}, \tag{A.8}$$

with law of motion given by:

$$N_t^C = N_{t-1}^C + \Psi_t^C - T_t^C \tag{A.9}$$

where Ψ_t^C are central bank's profits:

$$\Psi_t^C = i_{t-1}(N_{t-1}^C + M_{t-1}^C) + (r_t - i_{t-1})Q_{t-1}D_{t-1}^C. \qquad \text{(A.10)}$$

Combining (A.8), (A.9) and (A.10), the central bank's flow budget constraint follows:

$$Q_t D_t^C + \frac{B_t^C}{1+i_t} - M_t^C - \frac{X_t^C}{1+i_t} = (1-\varkappa_t)(1+\delta Q_t)D_{t-1}^C + B_{t-1}^C - X_{t-1}^C - M_{t-1}^C - T_t^C,$$

given initial conditions $D_{t_0-1}^C, B_{t_0-1}^C, X_{t_0-1}^C, M_{t_0-1}^C$ all equal to zero.

A.4 Equilibrium

Equilibrium in the goods market implies that

$$c_t = y,$$

at each time $t \geq t_0$ while equilibrium in the asset markets that

$$B_t + B_t^C = B_t^F,$$

$$M_t = M_t^C,$$

$$X_t = X_t^C,$$

$$D_t + D_t^C = 0.$$

A.5 Equilibrium conditions

I now characterize in a compact way the equilibrium conditions of the model.

The Fisher's equation follows from the Euler equation (A.5) using equilibrium in the goods market

$$1 + i_t = \frac{1}{\beta}\frac{P_{t+1}}{P_t}, \qquad \text{(A.11)}$$

while the equilibrium price of long-term securities is:

$$Q_t = \sum_{T=t}^{\infty} \delta^{T-t}\beta^{T+1-t}\left(\frac{P_t}{P_{T+1}}\right)\prod_{j=t+1}^{T+1}(1-\varkappa_j). \qquad \text{(A.12)}$$

Demand of real money balances is given by

$$\frac{M_t}{P_t} \geq L(y, i_t) \tag{A.13}$$

with

$$i_t \geq 0 \tag{A.14}$$

and the complementary slackness condition

$$i_t \left[\frac{M_t}{P_t} - L(y, i_t) \right] = 0. \tag{A.15}$$

The household's transversality condition can be simplified to

$$\lim_{T \to \infty} \left\{ \beta^{T-t_0} \left(\frac{P_{t_0}}{P_T} \right) \left(M_T + \frac{B_T + X_T}{1 + i_T} + Q_T D_T \right) \right\} = 0, \tag{A.16}$$

while the bound (A.4) can be written as

$$\sum_{T=t_0}^{\infty} \beta^{T-t_0} \left[y + \frac{i_T}{1 + i_T} L(y, i_T) \right] < \infty$$

which is naturally satisfied.

The flow budget constraints of treasury and central bank are respectively

$$\frac{B_t^F}{1 + i_t} = B_{t-1}^F - T_t^F - T_t^C, \tag{A.17}$$

$$Q_t D_t^C + \frac{B_t^C}{1 + i_t} - M_t^C - \frac{X_t^C}{1 + i_t} = (1 - \varkappa_t)(1 + \delta Q_t) D_{t-1}^C + B_{t-1}^C - X_{t-1}^C - M_{t-1}^C - T_t^C, \tag{A.18}$$

while equilibrium in the securities market closes the model

$$B_t + B_t^C = B_t^F, \tag{A.19}$$

$$M_t = M_t^C, \tag{A.20}$$

$$X_t = X_t^C, \tag{A.21}$$

$$D_t + D_t^C = 0. \tag{A.22}$$

A rational-expectations equilibrium is a collection of processes $\{P_t, i_t, M_t, Q_t, T_t^F, T_t^C, B_t, B_t^C, B_t^F, D_t^C, X_t\}_{t=t_0}^{\infty}$ that satisfy (A.11)-(A.19) at each date $t \geq t_0$ given (A.20)-(A.22) and initial conditions M_{t_0-1}, $B_{t_0-1}^C$, $B_{t_0-1}^F$, D_{t_0-1}, X_{t_0-1} all equal to zero. Since (A.16) is a bound and considering the complementary slackness condition (A.15), there are five degrees of freedom to specify the monetary/fiscal policy regime.