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Optimal Bank Capital Requirements: An Asymmetric Information Perspective*

Simone Berardi† Alessandra Marcelletti ‡

Abstract

The issue on the amount of capital banks should hold has pushed back the debate on top of policymakers’ agenda. Literature on this field mainly focuses on how to prevent banks from gaming risk-weighted capital requirements. The analysis has provided different types of solutions, such as the introduction of penalties and complementary use of risk-sensitive capital requirements and leverage ratio. Although the majority of theoretical papers rely on an asymmetric information framework, only one source of asymmetry is taken into account. The paper fills this gap by studying how to implement a socially optimal regulation scheme that simultaneously faces moral hazard and adverse selection problems. Including both sources of asymmetry is crucial because of the supervisor’s inability to distinguish between risk profiles and misconduct (risk-shifting behavior) of banks.

JEL classification: G21, G28, D81.
Keywords: bank capital requirements; bank regulation; moral hazard; adverse selection

1 Introduction

The development of the financial sector has traditionally been indicated as a key ingredient of economic growth, a catalyst to increase firm productivity and a way to foster entrepreneurship. Nevertheless, the quick expansion of the financial sector in recent decades has been associated with increased economic instability and fragility, as well as higher systemic vulnerability.

The debate on the socially optimal banking capital regulation scheme has recently drawn the attention of policymakers and scholars. Indeed, the global financial crisis has highlighted the need to review the regulatory framework governing the banking sector, which laid the groundwork that led to the development of the Basel III regulatory framework (Hannoun, 2010; Härle et al., 2010; Slovik and Cournède, 2011). Notwithstanding the substantial improvements in terms of stability granted by the new financial regulation, the banking system continues to show a high level of fragility. These vulnerabilities make the market unstable and prone to the possibility of speculative attacks; this shows that there is a need for continuous monitoring and reassessment of the regulatory framework, taking into account that market actors need stability. Currently, a growing debate about the revision of the Basel III framework towards Basel IV

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is coming to light, but it is now unlikely that a feasible solution would imply a completely new regulatory framework (Blundell-Wignall and Atkinson, 2010). A possible response to instability could be found in increasing capital requirements (Basten and Koch, 2015; Allen et al., 2012), since it can reduce risk-taking incentives, albeit Hellmann et al. (2000) claim that the overall effect of higher capital requirements is ambiguous. This is due to two opposite effects of the measure in a reduced form model of competition in deposit markets: capital at risk and franchise value. Once the increase in capital requirements takes place, the former induces banks to take on more risky assets, while the latter decreases the payoff associated with prudent investment. On the other hand, Repullo (2004) proves that in a dynamic model of imperfect competition, higher capital requirements reduce a bank’s incentive to gamble.

The central topic in the debate about the best regulatory scheme concerns the strength (ability) of the different capital requirements in reducing the opaqueness of the bank’s portfolio, and thus their misconduct. To solve the problem related to the evaluation of the riskiness of the bank, Basel II introduced the internal ratings based approach (IRB approach, henceforth), where the bank, through internal calculation, determines the risky parameters of its credit portfolio. Even though this regulatory framework is expected to align the internal risk assessments to the regulatory requirements, it leaves room for gaming the risk-based requirement (Jones, 2000). Indeed, a regulatory framework based on risk-weighted assets with the IRB approach implicitly assumes that the bank will truthfully reveal the riskiness of its portfolio, generating a moral hazard: since the choice over portfolio composition is observable only by banks, they can deviate from the announced risky parameter and direct resources towards low quality activities (Nielsen and Weinrich, 2016; Repullo, 2004). For this reason, the analysis of how to incentivize banks to report the true riskiness of their assets has been the object of analysis, above all after the implementation of Basel II (see Prescott, 2004, among others).

The ongoing studies on capital requirements suggest the adoption of a multi-polar regulation, i.e. risk-weighted assets with leverage ratio on top (see Estrella et al., 2000; Rugemintwari, 2011; Wu and Zhao, 2016; BCBS, 2014). Haldane et al. (2015) agrees with the Basel Committee on Banking Supervision BCBS (2010) that leverage ratio is a safeguard against the temptation of gaming risk based requirements. The analysis carried out by Dermine (2015) adds an important advantage to the adoption of unweighted capital requirements: during economically prosperous periods, it limits the probability of a bank run if there is imperfect information about loan losses. In a different framework, Jarrow (2013) gives two important justifications for the adoption of leverage ratio in capital adequacy rules: easier comparability across firms and more intuitiveness.

The welfare implication of adding leverage ratio on top of risk-weighted assets has been studied by Kiema and Jokivuolle (2014). Following the analysis of Repullo and Suarez (2004) and generalizing their results, they find that the overall effect of leverage ratio requirements on the incentive the bank might have to hold low-risk rather than high risk loans, depends on the type of equilibrium, i.e. on the impact of leverage ratio on loan interest rates. Indeed, if the bank is specialized in low-risk loans, it could change the composition by adding high-risk loans, whereas the leverage ratio increases the funding cost. Even if this paper offers an analysis of the welfare implication of capital requirements, it does not consider the possible asymmetric information problem between bank and regulator.

Recently, the strand of literature on the complementary use of risk-sensitive capital requirements and leverage ratio within an asymmetric information framework has been enforced

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1Repullo and Suarez (2004) analyze the possible consequences of loan pricing in an IRB approach and find that, under this regime, banks will tend to specialize in low-risk or high-risk lending.
by Wu and Zhao (2016). They develop a theoretical model of adverse selection with two risk-neutral maximizing agents, i.e. a bank and a regulator, where the former supports a cost of screening and a reputation cost, which may vary according to the different states of the economy. In this framework, they conclude that, in an endogenous framework of asset portfolio composition, both monitoring tools should be implemented by regulators, even if the optimal amount of leverage ratio depends on the probability of incurring in high (low) reputation cost. Their work is close to a study by Blum (2008) who, in an adverse selection framework with two different type of banks (risky and safe), demonstrates that the leverage ratio requirement, inversely related to the supervisory power of the regulator, can play the role of the penalty and can induce banks to have a truth-telling behavior.

The idea of introducing a penalty to encourage truthful behavior is due to the paper by Marshall and Prescott (2001). In order to control risk-taking behavior, they study state-contingent fines in a moral hazard framework, whereas bank quality is publicly observable. Their results demonstrate that state-contingent fines on banks that produce extremely high returns is socially optimal. However, since this result may lead to the conclusion that fines may be imposed on high quality banks, eventually deterring innovation, they address this issue by adding unobservable heterogeneity to their previous model. In this framework, Marshall and Prescott (2006), following up on the model of Marshall and Prescott (2001), demonstrate that the introduction of a state-contingent fine induces banks to undertake the socially optimal screening effort and deters risk-taking by low-quality banks. Kowalik (2011) further explores the possible benefit of imposing a penalty on the bank to avoid misreporting, in an asymmetric information framework, where the risk profile is the private information of the bank. By comparing recapitalization, downsizing, and closure of fines as the available supervisory tools to maximize social welfare, he concludes that risk-sensitive capital requirements makes the society better off when combined with recapitalization and fines.

In this paper we depart from the standard profit-maximization problem that encompasses only the banking sector perspective and we cast our model in a more vast framework, taking into account the system-wide impact of financial regulation. This model is in line with the recently emerged conceptional framework which is related to public goods, among which financial stability stands as a crucial example at a global level. Our aim is to identify the regulatory framework (leverage ratio or risk-weighted asset) that improves the stability of the financial system, combining moral hazard and the adverse selection framework. We show that in this context of incomplete information, taking into account the financial stability preservation into the optimal capital level problem suggests different solutions with respect to the standard case.

Indeed, despite the majority of the papers on capital requirements focused on the asymmetric information problem (see VanHoose, 2007, for a literature review), a minor strand of literature accounts for the moral hazard incentive and adverse selection simultaneously\(^2\), above all in a social welfare scenario. From a technical point of view, in this paper we follow the framework introduced by Laffont (1995). In his paper, he develops a theoretical model with both sources of asymmetric information, studying the optimal regulation scheme that the regulator may introduce in a natural monopoly to reduce environmental risk. Hence, the paper focuses on the possible incentive the regulator may provide to induce safety care, i.e. giving a rent to the most efficient firm and introducing a penalty for increasing safety. In the present paper, we extend this result by modelling the fines as contingent to each possible profit the

\[^2\]Giammarino et al. (1993), to analyze the possible risk-shifting behavior of banks, deal with both sources of asymmetric information in the context of deposit insurance schemes. However, although this paper introduces some social costs (bank failure, managerial cost, and the cost of government involvement in the banking sector) for the bank and the regulator, it does not compare the different possible capital requirements.
bank can gain from its misbehavior, i.e. from lying about the asset’s quality. Penalties aim to punish banks for the costs to society caused by misconduct and to discourage the misconduct itself in the future, and it can be in the form of monetary sanctions or business restrictions. Similar principles apply to the penalties for misconduct both in the European Union and in the United States. European Union fines are predominantly a result of mis-selling of guaranteed investment products and market manipulation. Legacy costs related to misconduct represent a relevant threat to bank profitability and capital ratios. Direct fines imposed by regulators do not represent the whole spectrum of misconduct costs. Indeed, banks bear the costs of fees for external consultants and the connected legal departments’ activity. Moreover, banks incur reputational costs, which are detrimental to future sales and funding conditions. Imposing this kind of penalty on financial firms may also increase public confidence in the banking sector, if they are seen as positive change drivers (ESRB, 2015).

The contribution of this paper to the existing literature is threefold: i) we provide evidence that the risk-weighted asset simultaneously solves the asymmetric information problems; ii) we demonstrate that uni-polar regulation in the banking sector allows the regulator to offset financial stability risk; and iii) we show the potential effectiveness of misconduct fines. In particular, from a policy perspective, this work suggests that the supervisory authority should ensure that banks adopt a capital strategy that guarantees financial stability by imposing fines if financial institutions behavior triggers financial turmoil.

2 General Framework

Consider an economy populated by two risk-neutral agents: a representative profit-maximizing oriented bank and a regulator. The goal of the regulator is to design the optimal regulation scheme that improves financial stability, considered here as a public good for the economy (Crockett, 1997), with a social value $S$ and maintenance cost $C$. We define the cost as $C = \beta - e_1$, where $\beta$ is the representative strand of the bank\(^3\) and $e_1$ is the screening effort of the bank. The latter, not observable by the regulator, is such that, with total assets normalized to 1, the proportion of the safe asset is $e_1$ and the proportion of the risky project is $1 - e_1$ (Wu and Zhao, 2016). The effort $e_1$ determines the composition of the bank’s portfolio because the screening activity enables the bank to better collect information about the counterparts. Hence, if the screening effort takes place, the bank will stipulate a riskless contract and will hold a greater amount of safe assets. Screening is costly for the bank and we assume that the cost function\(^4\) is $e_1^2m/2$.

The bank finances itself with capital $k$ and deposit $(1 - k)$, where the cost of capital $\delta$ is such that $\delta > 0$, which means that capital is more costly than the deposit (e.g. Allen, Carletti and Marquez, 2015; Vazquez and Federico, 2015). The idea behind this assumption is that, if the shareholders have to increase the bank’s capital, they will ask for a “premium” $\delta > 0$ to compensate for the opportunity cost of not investing in other projects that would yield a return of at least $\delta$. On the other hand, the depositors, looking at the certain return on their investments, are willing to accept a lower return. The deposits are fully insured by the regulator, with insurance premiums normalized to 0 and the interest rate to 0\(^5\).

\(^3\)It can be considered as the monopoly efficiency, where the higher the efficiency level, the higher the cost to the society, because of the stronger resulting bargaining power of the representative bank.

\(^4\)The cost function is increasing and convex for the volume of safe assets that they screen.

\(^5\)A justification for the assumption that the interest rate on the deposit is zero is that deposits are fully ensured by the regulator and depositors thus face no risk from the investment.
The bank can invest in safe and/or in risky assets: the safe asset has a constant return \( y \), while the return on the risky asset, \( \tilde{y} \), is distributed according to the following rule, where \( \tilde{y} \geq y \):

\[
\begin{cases}
  Y & \text{with probability } \theta \\
  0 & \text{with probability } (1 - \theta)
\end{cases}
\]  

(1)

The two-point distribution of the gross rate of return, and the introduction of financial disarray costs that the bank has to pay if \( \tilde{y} = 0 \), allows us to avoid a corner solution with infinite risk (Blum, 1999). In other words, although \( \tilde{y} \geq y \) and the bank is risk-neutral, we omit the possibility that the bank chooses only risky assets by assuming that, after some point, a further increase in risk leads to a decrease in expected return.

The bad state of the world, i.e. \( \tilde{y} = 0 \), creates the risk of financial disarray\(^6\), which imposes a cost \( B \) on society (financial stability is no longer verified, there can be negative externalities associated with contagion distortion, and the confidence in the financial market can be damaged). Furthermore, since the regulator aligns the capital requirement more closely to the risky parameter of the bank’s portfolio, the bank can influence the probability that a catastrophe occurs as follows. The bank reports to the regulator the risk credit portfolio and if it lies, the regulator cannot design the optimal regulation scheme. Reporting the actual parameter is costly for the bank because if the bank lies about the quality of its assets, it would lower the regulatory hurdle. \( e_2 \) denotes the revelation (of the risky parameter) cost, which assumes the value 1 if the bank truthfully reveals the asset quality and zero otherwise, and the bank’s choice of the effort level \( e_2 \) can increase the likelihood that the catastrophe (financial disarray) occurs\(^7\). Furthermore, when misconduct \( (e_2 = 0) \) is revealed, the bank faces penalty \( t \), which includes direct monetary sanctions and redress costs.

The timing of the model is:

- \( t = 1 \) the bank reveals\(^8\) the risky parameters of its credit portfolio based on its own internal calculation\(^9\) and the regulator imposes the capital regulation (leverage ratio or risk-weighted asset) according to the effort level that maximizes the social welfare, while nature chooses the parameter \( \beta \), which is the bank’s private information;

- \( t = 2 \) the bank chooses the level of effort \( e_1 \) and the regulator observes the cost \( C = \beta - e_1 \).

Formally, \( \psi(e_1, e_2) = \frac{e_1^2}{2} + e_2 \) denotes the disutility of the bank in exerting both efforts, where \( \psi(0) = 0 \), \( \psi'(e_1, e_2) > 0 \) and \( \psi''(e_1, e_2) > 0 \), i.e. the disutility is a positive and increasing function of the screening effort \( (e_1) \) and the truthful revelation of risky parameter \( (e_2) \).

The utility function of the bank \( U_b \) is composed of the utility of the gross revenue \( U(R, e_1) \), the cost \( t \), which is verified with probability \( (1 - \theta e_2) \) and the disutility \( \psi(e_1, e_2) \). The cost \( t \) is the cost that the bank supports in the event of financial disarray and could be interpreted as a misconduct fine imposed by the regulator, since the latter can catch the bank’s misreporting about the quality of its assets through periodical inspections.

---

\(^6\)Financial disarray may lead to bankruptcy when the return on the safe asset does not exceed the cost the bank can support.

\(^7\)This is formally accounted for in the model by adding the subscript \( e_2 \) to the probability \( \theta \).

\(^8\)We ensure truthful reporting by imposing \( e_2 = 1 \). As it will be clear from the following paragraph, this assumption, together with those connected to the revelation principle, clears out a possible strategic behaviour of bank.

\(^9\)The choice between leverage and risk-weighted asset does not depend on \( e_2 \). In this application of the IRB approach to time the relationship between the regulator and the bank, we follow Blum (2008).
To model the risk-neutrality of the bank, we adopt a linear utility function

\[ U_b = \theta_{e_2} \left[ ye_1 + Y(1 - e_1) - \delta k - \left( \frac{e_1^2 m}{2} + e_2 \right) \right] + (1 - \theta_{e_2}) \left[ ye_1 - \delta k - t(1 - e_1) - \left( \frac{e_1^2 m}{2} + e_2 \right) \right] \]

that can be rewritten in a more compact way as:

\[ U_b = ye_1 + \theta_{e_2} Y(1 - e_1) - \delta k - (1 - \theta_{e_2})(1 - e_1)t - \left( \frac{e_1^2 m}{2} + e_2 \right) = U(R, e_1) - (1 - \theta_{e_2})(e_2)(1 - e_1)t - \psi(e_1, e_2) \]

(2)

where the utility of the bank’s gross revenue is \( U(R, e_1) = ye_1 + \theta Y(1 - e_1) - \delta k \).

The capital amount \( k \) can take two values \( k = \{k_L; k_{RW}\} \), where \( k_L \) stands for the leverage ratio, and \( k_{RW} \) for the risk-weighted asset. In this paper, since total assets are normalized to 1, imposing a leverage requirement means that the regulator requires a minimum amount of capital, \( k_L \), while imposing a risk-weighted asset means requiring an amount of capital that allows the bank to compensate for loss, i.e. \( k_{RW} \leq (1 - e_1)\).

To select the optimal capital requirement, the regulator maximizes social welfare, balancing the benefit of offsetting risk, coherent with the bank’s profit maximization, and the opportunity cost of devoting public funds to detect lying banks and to maintain and/or improve financial stability.

Formally, we let \( V \) be social welfare, given by the social value of financial stability \( S \) and the bank’s utility \( U_b \), net of financial disarray cost for the society \( B \), and the opportunity cost of devoting public funds to the banking sector instead of the real economy. The latter also includes the surveillance cost, modelled as a positive and quadratic function\(^{10}\) of the risky assets of the bank, i.e. \( \frac{(1-e_1)^2}{2} \). To assess whether potential misconduct takes place, the regulator acts as a supervisor of the bank’s behavior, with the aim of increasing stakeholder confidence in the financial system and preventing future misconduct. The social welfare function is:

\[ V = S - (1 - \theta_{e_2})B - (1 + \lambda) \left( C - t(1 - \theta_{e_2})(1 - e_1) + \frac{(1-e_1)^2}{2} \right) + U_b \]

(3)

where \( (1 + \lambda) \) with \( \lambda > 0 \) is the social value of the public funds devoted to the banking sector.

To implement the contract, we first rewrite the social welfare as follows, taking into account that \( U_b = U(R, e_1) - \psi(e_1, e_2) - (1 - \theta)(1 - e_1)t \):

\[ V = S - (1 - \theta_{e_2})B - (1 + \lambda) \left( C - t(1 - \theta_{e_2})(1 - e_1) + \frac{(1-e_1)^2}{2} \right) + U_b \]

\[ = S - (1 - \theta_{e_2})B - (1 + \lambda) \left( C - U(R, e_1) + \psi(e_1, e_2) + U_b + \frac{(1-e_1)^2}{2} \right) + U_b \]

(4)

The regulator’s problem of improving financial stability is:

\(^{10}\)The surveillance cost function is modelled as a quadratic to capture the fact that it is an increasing function in the risky asset.
max \( e_1 \) \( S - (1 - \theta e_2) B - (1 + \lambda) \left( \beta - e_1 - U(R, e_1) + \psi(e_1, e_2) + \frac{(1-e_1)^2}{2} \right) - \lambda U_b \)

s.t. \( U(R, e_1) - (1 - \theta e_2)(1 - e_1) t - \left( \frac{e_1^2 m}{2} + 1 \right) \geq 0 \)  \hspace{1cm} (5)

2.1 Complete Information: Optimal Solution

Under complete information, the regulator knows the efficiency parameter \( \beta \), the screening effort of the bank, and will simply maximize (5) to find the optimal amount of assets under regulation. Furthermore, the participation constraint holds with equality \((U_b = 0)\), and the bank will always truthfully reveal the risky parameter of the credit portfolio, i.e. \( e_2 = 1 \). Hence, to implement a socially optimal regulation scheme, the regulator faces the following maximization problem:

max \( e_1 \) \( S - (1 - \theta) B - (1 + \lambda) \left( \beta - e_1 - U(R, e_1) + \psi(e_1, 1) + \frac{(1-e_1)^2}{2} \right) - \lambda U_b \)

s.t. \( ye_1 + \theta Y(1 - e_1) - \delta k - (1 - \theta)(t)(1 - e_1) - \left( \frac{e_1^2 m}{2} + 1 \right) = 0 \)  \hspace{1cm} (6)

where the first order condition reads as:

\[
\frac{dV}{de_1} = (1 + \lambda) + (1 + \lambda) \frac{dU(R, e_1)}{de_1} - (1 + \lambda) \psi'(e_1, e_2) + (1 + \lambda)(1 - e_1) \quad (7)
\]

By calculating \( \frac{dU(R, e_1)}{de_1} \) under both capital requirements and setting the first derivative to zero, we can determine the lowest marginal disutility coherent with the maximization of social welfare. It is worth noting that, under complete information, we suppress the subscript \( e_2 \) on \( \theta e_2 \), since we assume that the bank always reveals the truth. Under complete information, the optimal marginal disutility is: \( \psi'(e_1, e_2)_L = 1 + (1 - e_1) + y - \theta Y + (1 - \theta) t - e_1 m \) under leverage ratio and \( \psi'(e_1, e_2)_RW = 1 + (1 - e_1) + y - \theta Y + (1 - \theta) t - e_1 m + \delta y \) under the risk-weighted asset. Thus, the following proposition summarizes the result.

**Result 1:** Under complete information, the bank is better off (has a lower marginal disutility) under the leverage ratio requirement.

A possible explanation is that since holding a higher amount of safe assets is costly for the bank, if the regulator knows the efficiency and the bank truthfully reveals its risky parameter, there will be no incentive to constrain the bank’s portfolio composition. Furthermore, the leverage ratio is simple to implement and monitor, and it reduces regulatory arbitrage. On the other hand, if the regulator has complete information about the bank’s risk profile, the limitations of the leverage ratio as a monitoring tool will diminish. Hence, the best model for aligning the bank’s interest and regulatory requirements in a social welfare perspective is leverage ratio.
### 2.2 Definition of Effort under Complete Information

Before proceeding with the identification of which is the second best outcome of this model, we firstly need the definition of low and high effort that the bank can undertake under both regimes. To this aim, we assume that the bank maximizes its utility function under both leverage and risk-weighted assets and identifies its optimal screening effort level $e_1$, keeping in mind that the amount of capital $k$ can assume two values, $k = \{k_L, k_{RW}\}$. We further assume that in this phase, the bank will always reveal the truth ($e_2 = 1$). It is worth recalling that a wrong report implies a suboptimal regulatory scheme that, by ruining financial stability, might induce financial disarray\(^\text{11}\).

The utility function of the bank under the leverage ratio requirement is:

$$U_b = ye_1 + \theta Y(1 - e_1) - \delta k_L - (1 - \theta)(1 - e_1) - \left(\frac{e_1^2 m + 1}{2}\right)$$  \hspace{1cm} (8)

where $\frac{e_1^2 m + 1}{2} = \psi(e_1, e_2)$, and $k = k_L$, with leverage ratio as explained in Section 2.

Thus, the optimal amount of safe assets the bank will hold under leverage ratio corresponds to the amount of safe asset that maximizes the expected profit, given the capital requirement restriction. For this reason, we derive (8) with respect to $e_1$,

$$\frac{dU_b}{de_1} = y - Y \theta + (1 - \theta)t - e_1 m$$  \hspace{1cm} (9)

and, setting the FOC equal to zero, we rearrange to obtain:

$$e_{LR}^* = \frac{y - \theta Y}{m} + \frac{(1 - \theta)t}{m}$$  \hspace{1cm} (10)

which is the optimum level of $e_1$ the bank will hold if the regulator chooses leverage ratio as a regulatory tool. In particular, as equation (10) shows, the screening effort is positive and increasing for the penalty cost $t$ and for the return of the safe asset\(^\text{12}\).

To derive the optimal amount of safe assets under the risk-weighted capital requirement, we simply state the bank’s utility function as follows, recalling that the amount of capital required under risk-weighted capital is: $k_{RW} = k_{RW} \leq 1 - e_1 y$. Hence,

$$U_b = ye_1 + \theta Y(1 - e_1) - (1 - e_1 y) - (1 - \theta)(1 - e_1) + t - \left(\frac{e_1^2 m + 1}{2}\right)$$  \hspace{1cm} (11)

The optimum amount of safe assets the bank will hold in the risk-weighted capital regulation framework is obtained by maximizing (11) with respect to $e_1$:

$$\frac{dU_b}{de_1} = y - \theta Y + \delta y + (1 - \theta)(-1)(-t) - e_1 m$$  \hspace{1cm} (12)

Setting the derivative equal to zero, we obtain:

$$e_{RW}^* = \frac{\delta y}{m} + \frac{y - \theta Y}{m} + \frac{(1 - \theta)t}{m}$$  \hspace{1cm} (13)

\(^\text{11}\)As will be clear from the following paragraph, since we model the financial disarray as an event that also occurs as a consequence of the bank’s misbehaviour, the bank will pay a fine that is defined according to the revelation principle.

\(^\text{12}\)In order to have non-negative values for the amount of safe assets the bank will hold, we set $e_{LR}^* \geq 0$, implying that $(1 - \theta)t \geq \theta Y - y$, i.e. the penalty the bank will pay if financial distress occurs should be at least equal to the earning difference between risky and safe assets.
which is the optimal amount of safe assets the bank will hold if the regulator imposes risk-weighted asset capital requirements. Equation (13) implies that the screening effort under this framework depends on the safe return of investment, the penalty, and the cost of capital. Since $e_{RW}^*$ can be rewritten as $e_L^* + \frac{\delta y}{m}$ where $\frac{\delta y}{m}$ is always positive, the bank in the risk-weighted asset capital requirement framework will always hold a greater amount of safe assets than in the case of leverage ratio. In particular, since we assume that capital is more costly than deposits, Result 2 can be formalized as follows.

**Result 2:** If capital is costly, risk-weighted capital requirements incentivize banks to increase the amount of safe assets (i.e. to exert more screening effort) more than they would under a leverage ratio regime.

Given the result in Result 2, we can conclude that $e_{LR}^*$ can be formally denoted as the low screening effort ($e_1$) and $e_{RW}^*$ as the high screening effort ($\bar{e}_1$). The intuition behind this is that the effort required of the bank in the risk-weighted asset framework is always greater than the effort with leverage ratio since, according to the definition of screening effort $e_1$, the former requires holding a higher amount of safe assets.

### 2.3 Incomplete Information

Under complete information, the regulator knows the efficiency parameter and asset quality, with the bank truthfully revealing the quality of the asset. Under incomplete information, the regulator does not know $\beta, e_1, e_2$, but it observes the cost $C$ ex-post and the realization of $\tilde{y}$. The regulator wants to encourage the bank to reveal the truth, implying that the regulator wants to implement an equilibrium with $e_2 = 1$ since the cost of having a suboptimal regulation scheme, as well as the cost of having an unstable banking sector, is high. The regulator wants the bank to truthfully reveal its efficiency type $\beta$ that can assume two values, $\bar{\beta}$ or $\tilde{\beta}$, with $\bar{\beta} > \beta$ and $v = \text{Prob}\{\beta = \bar{\beta}\}$. Thus, the following (rationality) constraints should be satisfied for the identification of the (second) best regulatory scheme:

\[
U_b(\beta, \beta, 1) \geq U_b(\bar{\beta}, \bar{\beta}, 1) \quad (14)
\]

\[
U_b(\beta, \beta, 1) \geq U_b(\beta, \bar{\beta}, 0) \quad (15)
\]

\[
U_b(\beta, \beta, 1) \geq U_b(\bar{\beta}, \bar{\beta}, 0) \quad (16)
\]

\[
U_b(\beta, \beta, 1) \geq U_b(\bar{\beta}, \beta, 1) \quad (17)
\]

\[
U_b(\beta, \beta, 1) \geq U_b(\bar{\beta}, \bar{\beta}, 0) \quad (18)
\]

\[
U_b(\bar{\beta}, \bar{\beta}, 1) \geq U_b(\bar{\beta}, \bar{\beta}, 0) \quad (19)
\]

as well as the participation constraints:

\[
U_b(\beta, \beta, 1) \geq 0 \quad (20)
\]

\[
U_b(\bar{\beta}, \bar{\beta}, 1) \geq 0 \quad (21)
\]

We start from the incentive problem\(^{13}\), i.e. we look at constraints 14, 17, 20 and 21, where we know that the binding constraints are 14 and 21. Indeed, it is well known that, in this kind of

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\(^{13}\)We will discuss the other constraints, by looking at how the penalty can be imposed to incentivize bank to truthfully reveal the risky parameters, i.e. we will look at constraints 15, 16, 18 and 19
problem, the rationality constraints of the inefficient type and the participation constraint of the efficient type hold with equality, meaning that \( U_\theta(e_1, e_1) = U_\theta(e_1, e_1) = 0 \). This prevents banks with high risk portfolios from mimicking low risk profiles to allocate a suboptimal level of capital\(^{14}\). Under incomplete information, the bank can announce different types of \( \tilde{\beta} \), and it will consequently pay a cost \( t^*(\tilde{\beta}) \), if the financial disarray occurs, and a cost \( t^m(\tilde{\beta}) \), if there is no financial disarray\(^{15}\). In this light, we rewrite the utility function of the bank as follows:

\[
U(\beta, \tilde{\beta}, e_2) = U(R, \tilde{\beta}) + t_{mc}(\tilde{\beta})(1 - \tilde{e}_1) - (1 - \theta e_2) t^*(\tilde{\beta})(1 - \tilde{e}_1) - \psi(\beta - \tilde{\beta} + \tilde{e}_1 + e_2)
\] (22)

As shown in equation 22, under incomplete information, untruthful behavior affects the bank’s utility function, above all on the expenditure side. Indeed, the penalty the bank should pay, whether or not the financial disarray occurs, is proportional to the amount of risky assets the bank holds and the catastrophe’s probability. It is worth recalling that this risk is influenced by the behavior of the bank: if it does not report the true credit risk portfolio, it could lower the regulatory hurdle but also increase the probability of financial disarray.

Under incomplete information, the social planner has to also deal with the possibility of the bank’s untruthful behavior when designing the optimal regulation scheme. In this light, the social welfare can be rewritten as:

\[
V = v[S - (1 - \theta e_2)B - (1 + \lambda)(\beta - \xi_1 - U(R, \xi_1) + \psi(\xi_1, e_2) + \frac{(1 - \xi_1)^2}{2}) - \lambda U_b] + \lambda v(Y - e_1 + e_2) (23)
\]

Since the revelation effort is unknown to the regulator, the goal of the regulator is to implement a contract, i.e. a regulatory scheme that optimizes the social welfare function. To do so, it has to identify the effort to require of the bank. For this reason, it maximizes the social welfare function under both scenarios. Under the leverage ratio framework, by optimizing the social welfare function as in 23, we obtain the following optimal marginal disutility function of the bank\(^{16}\):

\[
\psi'(\xi_1 + 1) = 1 + (1 - \xi_1) + y - \theta Y + (1 - \theta) t - \xi_1 m
\] (24)

while, under the risk-weighted asset regulatory framework, the optimal marginal disutility\(^{17}\) is:

\[
\psi'(\tilde{e}_1 + 1) = 1 + (1 - \tilde{e}_1) + y - \theta Y + (1 - \theta) t - \tilde{e}_1 m + \delta y - \frac{\lambda v}{(1 + \lambda)(1 - v)} dU(\beta, \tilde{\beta}, 1)\] (25)

To evaluate the effort the regulator requires of the bank that makes the society better off, we compare the marginal disutility given the two-level effort under the leverage ratio and risk-weighted assets. It is possible to show\(^{18}\) that \( \psi'(\xi_1 + 1) > \psi'(\tilde{e}_1 + 1) \) if:

\[
\frac{\delta y}{m} + \frac{\lambda v}{(1 + \lambda)(1 - v)} \frac{dU(\beta, \tilde{\beta}, 1)}{d\tilde{e}_1} > 0
\] (26)

\(^{14}\)As common in literature, the incentive compatibility constraint for the good type and the participation constraint for the bad type are binding.

\(^{15}\)It is worth recalling that financial disarray can occur for two reasons: a bad realization of the risky assets, i.e. \( \tilde{y} = 0 \), and a suboptimal design of the capital requirements due to the incorrect definition of the risk parameter.

\(^{16}\)See Appendix for technical details.

\(^{17}\)See Appendix for technical details.

\(^{18}\)See Appendix for technical details.
Since both terms on the left side of the equation are greater than zero, the condition in equation (26) is always met. We can thus formalize the following:

**Result 3:** Under incomplete information, risk-weighted asset always implies the lowest marginal disutility for the bank.

This result implies that the second best option for the society is always to implement the risk-weighted asset instead of the leverage ratio. This result could be explained by different aspects of the incomplete information scenario: the regulator does not know the credit portfolio’s composition, nor the bank’s type, and the bank can undertake hidden actions. These factors imply that there is the risk of a shift in the bank balance sheets toward excessive exposure to risky assets, as well as the risk of untruthful behavior. For this reason, under incomplete information, the risk-weighted asset framework makes the society better off.

At this point, since the optimal contract is risk-weighted assets, the regulator has to design the optimal incentive scheme that prevents the bank’s untruthful behavior. This is equivalent to checking if the constraints in 15, 16, 18 and 19 are verified without additional costs to the society. To this aim, we rewrite the rationality constraints that ensures that \( e_2 = 1 \) under both regimes.

\[
\ell^c - \ell^{nc} \geq \frac{\psi(e_1 + 1) - \psi(\bar{e}_1)}{1 - e_1}(\theta_1 - \theta_0) \tag{15'}
\]

\[
\bar{\ell}^c - \bar{\ell}^{nc} \geq \frac{\psi(\bar{e}_1 + 1) - \psi(e_1)}{1 - e_1}(\theta_1 - \theta_0) \tag{18'}
\]

To incentivize the bank to not lie about its asset quality, the penalty is such that, in the case of financial disarray, the bank will pay an extra. As highlighted in equation 16’ and 19’, this amount is proportional to the earnings due to the lower disutility the bank will get by lying about the risk parameters, and since \( \theta(1) > \theta(0) \), the penalty is greater in the case of misconduct.

Furthermore, to verify that \( U_b(\bar{\beta}, \bar{\beta}, 1) \geq U_b(\bar{\beta}, \bar{\beta}, 0) \) and \( U_b(\bar{\beta}, \bar{\beta}, 1) \geq U_b(\bar{\beta}, \bar{\beta}, 0) \), i.e. to ensure that the bank will not lie about the efficiency type and shift their risk profile, the following conditions should be satisfied:

\[
\bar{\ell}^c - \ell^c \geq \theta_1(1 - e_1)(\ell^{nc} - \ell^c) - \theta_0(1 - \bar{e}_1)(\bar{\ell}^{nc} - \bar{\ell}^c) + \psi(e_1 + 1) - \psi(\bar{e}_1 - \Delta \beta) + a \tag{16'}
\]

\[
\ell^c - \bar{\ell}^c \geq \theta_1(1 - \bar{e}_1)(\ell^{nc} - \bar{\ell}^c) - \theta_0(1 - e_1)(\ell^{nc} - \ell^c) + \psi(\bar{e}_1 + 1) - \psi(e_1 + \Delta \beta) + b \tag{19'}
\]

where \( \Delta \beta = \bar{\beta} - \beta, a = U(R, \bar{e}_1) - U(R, e_1) + \bar{\ell}^c \bar{e}_1 - \ell^c e_1 \), and \( b = \ell^c e_1 + \bar{\ell}^c \bar{e}_1 - U(R, \bar{e}_1) - U(R, e_1) \).

That is, to incentive the bank to declare the real efficiency level and the true risk parameters \( e_2 = 1 \), the regulator imposes a penalty that is greater than all the possible earnings derived from lying: the differences in disutility and gross profit, as well as the eventual fine if financial disarray occurs. Hence, the following arises:

\[\text{See Appendix for technical details.}\]
Result 4: Financial stability can be improved and/or maintained if the regulator implements an optimal risk-weighted asset capital requirement regulation scheme, with the misconduct penalty set such that the bank will truthfully reveal the risk parameters of its credit portfolio.

These constraints allow us to cast the model in a social welfare perspective. Indeed, the second best regulatory framework imposed by the social planner to increase financial stability is such that the bank internalizes the risk of financial disarray by paying a fine contingent on the financial market’s condition. These penalties ensure that the bank will truthfully reveal the credit risk parameters, going beyond the limitations of the risk-weighted asset regulatory tool.

3 Policy Implications

Basel III principles were primarily intended to focus attention on the numerator of the capital adequacy ratio, giving importance to both the quality and quantity of a bank’s capital (Repullo and Saurina Salas, 2011). Of course, the denominator was also considered absolutely relevant, but the focus was confined to specific areas, such as the risk weight on securitisations and counterparty credit risk.

Recently, the Basel Committee has concentrated more comprehensively on the denominator of the capital ratio, with growing attention on a bank’s risk-weighted credit, market, and operational risk exposures. The process is aimed at introducing a revised set of standardised mechanisms, also identifying constraints capable of controlling for the extent to which banks can reduce their capital requirements through the use of internal models. This is of paramount importance now, as the path towards a revision of the Basel III framework seems to be on a steep trajectory (Berger et al., 2016). The forefront of those who claim the need to deepen reflection about the regulatory architecture is composed of various instances, but the central theme is to identify homogeneous, standardized measures of risk as replacements for bank internal models for calculating capital requirements. These models potentially cover market, credit, interest rate, operational, sovereign, and step-in risk. Some analysts and financial regulators have suggested moving forward towards a simpler regulatory framework based on a leverage ratio. According to the proponents of this option, the simple leverage ratio would allow easier control by regulatory authorities and would limit the obstacles to the regulatory process inherent in the discretion of the banks in the assessment of the riskiness of different assets. It would also reduce the complexity embedded in the IRB approach.

The model presented in the paper shows that, in a complete information environment, the leverage ratio regulatory framework is considered superior by the bank. This implies that the internal ratings-based model does not align the bank’s interests. Although the need to make the regulatory framework less cumbersome is evident, it is crucial to note the regulators’ mission, which is to try to simultaneously ensure the soundness of the financial system and the stability of the whole economy. With this in mind, it is necessary to identify the mechanism that maximizes the overall social welfare, given the constraints that characterize the system, namely incomplete and imperfect information, together with moral hazard. In light of the previous considerations concerning the goal of the whole system is stability, this paper designs a theoretical framework that can be considered consistent with the reality of the current financial system.

The main result of this paper is that, under incomplete information, the risk-weighted
asset always implies the lowest marginal disutility for the bank. Implementing the optimal risk-weighted asset capital requirement regulation scheme, the regulator is able to improve the systemic stability. So, in order to ensure that the bank truthfully reveals the risk parameters of its credit portfolio, it is crucial to design proper and efficient incentive schemes based on penalties. In this way, the adoption the second best approach, i.e. the risk-weighted asset scheme, ensures the maximization of social welfare, given the constraints. This result corroborates the thesis that the supporters of the superiority of the risk-weighted asset scheme, who see in the approach based on leverage the risk of a shift in the composition of bank balance sheets toward excessive exposure in risky assets (even leading to corner solutions in the most extreme cases).

4 Concluding Remarks

In this paper we broaden the analysis about the optimal level of capital moving beyond the relationship between banks and supervisory authorities. Indeed, this issue has profound influences on financial stability, which is now recognized as global public good and cannot be excluded from this debate, especially in light of the persistent consequences of the financial crisis. However, until now most of the analysis in this field has not explored this aspect, while our paper tries to shed some light in this direction. More specifically, we focused on identifying the regulatory framework that improves the stability of the financial system, departing from the banking perspective in favour of a social welfare perspective. We consider and compare, from a social welfare point of view, two regulatory tools: the leverage ratio and the risk-weighted capital. Following the model introduced by Laffont (1995), we implemented a socially optimal regulation scheme that simultaneously deals with both sources of asymmetric information: moral hazard and adverse selection. The idea behind combining both asymmetries relies on the inability of the supervisor to distinguish between risk profiles and bank misconduct (risk-shifting behavior). As a result of the model, we conclude that the leverage ratio is the first best solution, while risk-weighted asset is second best. The intuition behind this result is that, in a symmetric information framework, there is no need to compell the bank’s portfolio composition. On the other hand, if the supervisor does not know the risk profile and efficiency type, it is socially optimal to impose a risk-weighted capital requirement. Indeed, it implies the lowest marginal disutility for the bank and ensures the maximization of the social welfare function, where financial stability is modelled as a public good for the whole economy. An important policy recommendation that could address the moral hazard and adverse selection problems at the same time is the use of penalties. As shown in our analysis, the gaming strategy and untruthful behavior might be discouraged by imposing fines on the bank. Indeed, if the penalty is contingent on the degree of financial stability and the bank’s possible earnings from lying, it will decrease the bank’s expected profit and lead the bank to undertake truthful behavior. Furthermore, by internalizing the cost of financial disarray, the bank is incentivized not to game the risk parameter.

Another possible policy tool would be a risk-weighted assets with leverage ratio on top, and we are going to model it in a future work.
Appendix: Mathematical Proofs

Proof of Result 3
To demonstrate that \( \psi'(\epsilon_1 + 1) > \psi'(\bar{e}_1 + 1) \), we proceed in two steps. First we compute the optimal marginal disutility under the two capital requirements and then we demonstrate that the disequality always holds.

In the first step, we start deriving (23) with respect to \( \epsilon_1 \), and thus obtain:

\[
\frac{dV}{d\epsilon_1} = v(1 + \lambda) + v(1 + \lambda)\frac{dU(R, \epsilon_1)}{d\epsilon_1} - v(1 + \lambda)\psi'(\epsilon_1 + 1) + v(1 + \lambda)(1 - \epsilon_1)
\]

Setting it equal to zero, we obtain:

\[
\psi'(\epsilon_1 + 1) = 1 + \frac{dU(R, \epsilon_1)}{d\epsilon_1} + (1 - \epsilon_1)
\]

Deriving (23) with respect to \( \bar{e}_1 \), recalling that \( U(\beta, \bar{\beta}, 1) \) can be rewritten as \( U(\beta, \bar{\beta}, 1) = U(R, \epsilon_1) - (1 - \theta)(1 - \epsilon_1) - \psi(\beta - C(\bar{\beta} + 1)) \), we obtain:

\[
\frac{dV}{d\bar{e}_1} = v\lambda \frac{dU(R, \bar{e}_1)}{d\bar{e}_1} - (1-v)(1+\lambda) + (1-v)(1-\lambda) \frac{dU(R, \bar{e}_1)}{\bar{e}_1} - (1-v)(1+\lambda)\psi'(\bar{e}_1 + 1) + (1-v)(1+\lambda)(1-\bar{e}_1)
\]

Setting the first derivative equal to zero and rearranging, we obtain:

\[
\psi'(\bar{e}_1 + 1) = 1 + \frac{dU(R, \bar{e}_1)}{\bar{e}_1} + (1 - \bar{e}_1) - \frac{\lambda v}{(1+\lambda)(1-v)} \frac{dU(\beta, \bar{\beta}, 1)}{d\bar{e}_1}
\]

Now, we can demonstrate that \( \psi'(\epsilon_1 + 1) > \psi'(\bar{e}_1 + 1) \). We equate equation (24) and equation (25):

\[
-\epsilon_1 - \epsilon_1 m > -\bar{e}_1 - \bar{e}_1 m + \delta y - \frac{\lambda v}{(1+\lambda)(1-v)} \frac{dU(\beta, \bar{\beta}, 1)}{d\bar{e}_1}
\]

Recalling that \( \epsilon_1 = e_1^* \) and \( \bar{e}_1 = e_{RW}^* = e_1^* + \frac{\delta y}{m} \), we obtain:

\[
-e_L - e_L m > -e_L - e_L m + \delta y - \frac{\lambda v}{(1+\lambda)(1-v)} \frac{dU(\beta, \bar{\beta}, 1)}{d\bar{e}_1}
\]

Simplifying and rearranging, we get the condition in (26):

\[
\frac{\delta y}{m} + \frac{\lambda v}{(1+\lambda)(1-v)} \frac{dU(\beta, \bar{\beta}, 1)}{d\bar{e}_1} > 0
\]

Since both terms on the left side of the disequality are greater than zero, the condition is always met. ■

Proof of Result 4
To obtain the constraints in 15’, 16’, 18’, and 19’ we start by substituting the bank’s utility function in the respective rationality constraints 15, 16, 18, and 19 as described in equation 22.
By denoting $t$ with $t(\beta)$, the constraint 15 can be rewritten as:

$$U(R, e_1) - \theta(1)\bar{\ell}^{nc} - (1 - \theta(1))\bar{\ell}^{c} - \psi(e_1 + 1) \geq U(R, e_1) - \theta(0)\bar{\ell}^{nc} - (1 - \theta(0))\bar{\ell}^{c} - \psi(e_1)$$

Rearranging:

$$\frac{(1 - e_1)(-\theta(1)\bar{\ell}^{nc} - (1 - \theta(1))\bar{\ell}^{c} + \theta(0)\bar{\ell}^{nc} + (1 - \theta(0))\bar{\ell}^{c})}{(1 - \theta(1) + \theta(0))} \geq \psi(e_1 + 1) - \psi(e_1)$$

$$\bar{\ell}^{nc}(\theta(0) - \theta(1)) + \bar{\ell}^{c}((\theta(1)) - \theta(0)) \geq \frac{\psi(e_1 + 1) - \psi(e_1)}{(1 - e_1)}$$

Hence,

$$\bar{\ell}^{c} - \bar{\ell}^{nc} \leq \frac{\psi(e_1 + 1) - \psi(e_1)}{(1 - e_1)(\theta(1) - \theta(0))}$$

Similarly, we can derive condition 18'. By denoting $\bar{t}$ with $t(\beta)$, constraint 18 can be rewritten as:

$$U(R, \bar{e}_1) - \theta(1)\bar{\ell}^{nc} - (1 - \theta(1))\bar{\ell}^{c} - \psi(e_1 + 1) \geq U(R, \bar{e}_1) - \theta(0)\bar{\ell}^{nc} - (1 - \theta(0))\bar{\ell}^{c} - \psi(e_1)$$

Rearranging:

$$\frac{(1 - e_1)(-\theta(1)\bar{\ell}^{nc} - (1 - \theta(1))\bar{\ell}^{c} + \theta(0)\bar{\ell}^{nc} + (1 - \theta(0))\bar{\ell}^{c})}{(1 - \theta(1) + \theta(0))} \geq \psi(e_1 + 1) - \psi(e_1)$$

$$\bar{\ell}^{nc}(\theta(0) - \theta(1)) + \bar{\ell}^{c}((\theta(1)) - \theta(0)) \geq \frac{\psi(e_1 + 1) - \psi(e_1)}{(1 - e_1)}$$

Hence,

$$\bar{\ell}^{c} - \bar{\ell}^{nc} \leq \frac{\psi(e_1 + 1) - \psi(e_1)}{(1 - e_1)(\theta(1) - \theta(0))}$$

Now, we derive the constraints in 16'. Firstly, we rewrite 16 as:

$$U(R, e_1) - \theta(1)\ell^{nc}(1 - e_1) - (1 - \theta(1))\ell^{c}(1 - e_1) - \psi(e_1 + 1) \geq U(R, e_1) - \theta(0)\ell^{nc}(1 - e_1) - (1 - \theta(0))\ell^{c}(1 - e_1) - \psi(e_1 - \Delta \beta)$$

Rearranging:

$$-\theta(1)\ell^{nc}(1 - e_1) + \theta(0)\ell^{nc}(1 - e_1) + (1 - \theta(0))\ell^{c}(1 - e_1) - (1 - \theta(1))\ell^{c}(1 - e_1) \geq U(R, e_1) - U(R, e_1) + \psi(e_1 + 1) - \psi(e_1 - \Delta \beta)$$

Rearranging the left hand side:

$$-\theta(1)\ell^{nc}(1 - e_1) + \theta(0)\ell^{nc}(1 - e_1) + \ell^{c} - \ell^{c}e_1 - \theta(0)\ell^{c}(1 - e_1) - \ell^{c} + \ell^{c}e_1 + \theta(1)\ell^{c}(1 - e_1)$$

That can be rewritten as:

$$\ell^{c} - \ell^{c} \geq \theta(1)(1 - e_1)(\ell^{nc} - \ell^{c}) - \theta(0)(1 - e_1)(\ell^{nc} - \ell^{c}) + \psi(e_1 + 1) - \psi(e_1 - \Delta \beta)$$

Hence,

$$\ell^{c} - \ell^{c} \geq \theta(1)(1 - e_1)(\ell^{nc} - \ell^{c}) - \theta(0)(1 - e_1)(\ell^{nc} - \ell^{c}) + \psi(e_1 + 1) - \psi(e_1 - \Delta \beta) + a$$

The constraints in 19' can be derived as follows. Firstly, we rewrite 19 as:

$$U(R, e_1) - \theta(1)\ell^{nc}(1 - e_1) - (1 - \theta(1))\ell^{c}(1 - e_1) - \psi(e_1 + 1) \geq U(R, e_1) - \theta(0)\ell^{nc}(1 - e_1) - (1 - \theta(0))\ell^{c}(1 - e_1) - \psi(e_1 + \Delta \beta)$$

Rearranging:

$$-\theta(1)\ell^{nc}(1 - e_1) - (1 - \theta(1))\ell^{c}(1 - e_1) + \theta(0)\ell^{nc}(1 - e_1) + (1 - \theta(0))\ell^{c}(1 - e_1) \geq U(R, e_1) - U(R, e_1) + \psi(e_1 + 1) - \psi(e_1 + \Delta \beta)$$

Rearranging the left hand side:

$$-\theta(1)\ell^{nc}(1 - e_1) - (1 - \theta(1))\ell^{c}(1 - e_1) + \ell^{c} - \ell^{c}e_1 - (1 - \theta(0))\ell^{c}(1 - e_1) - \ell^{c} + \ell^{c}e_1 + (1 - \theta(1))\ell^{c}(1 - e_1)$$

That can be rewritten as:

$$\ell^{c} - \ell^{c} \geq \theta(1)(1 - e_1)(\ell^{nc} - \ell^{c}) - \theta(0)(1 - e_1)(\ell^{nc} - \ell^{c}) + \psi(e_1 + 1) - \psi(e_1 + \Delta \beta) + a$$

Hence,

$$\ell^{c} - \ell^{c} \geq \theta(1)(1 - e_1)(\ell^{nc} - \ell^{c}) - \theta(0)(1 - e_1)(\ell^{nc} - \ell^{c}) + \psi(e_1 + 1) - \psi(e_1 + \Delta \beta) + a$$

The constraints in 19' can be derived as follows. Firstly, we rewrite 19 as:

$$U(R, e_1) - \theta(1)\ell^{nc}(1 - e_1) - (1 - \theta(1))\ell^{c}(1 - e_1) - \psi(e_1 + 1) \geq U(R, e_1) - \theta(0)\ell^{nc}(1 - e_1) - (1 - \theta(0))\ell^{c}(1 - e_1) - \psi(e_1 + \Delta \beta)$$

Rearranging:

$$-\theta(1)\ell^{nc}(1 - e_1) - (1 - \theta(1))\ell^{c}(1 - e_1) + \theta(0)\ell^{nc}(1 - e_1) + (1 - \theta(0))\ell^{c}(1 - e_1) \geq U(R, e_1) - U(R, e_1) + \psi(e_1 + 1) - \psi(e_1 + \Delta \beta)$$

Rearranging the left hand side:

$$-\theta(1)\ell^{nc}(1 - e_1) - (1 - \theta(1))\ell^{c}(1 - e_1) + \ell^{c} - \ell^{c}e_1 - (1 - \theta(0))\ell^{c}(1 - e_1) - \ell^{c} + \ell^{c}e_1 + (1 - \theta(1))\ell^{c}(1 - e_1)$$

That can be rewritten as:

$$\ell^{c} - \ell^{c} \geq \theta(1)(1 - e_1)(\ell^{nc} - \ell^{c}) - \theta(0)(1 - e_1)(\ell^{nc} - \ell^{c}) + \psi(e_1 + 1) - \psi(e_1 + \Delta \beta) + a$$

Hence,

$$\ell^{c} - \ell^{c} \geq \theta(1)(1 - e_1)(\ell^{nc} - \ell^{c}) - \theta(0)(1 - e_1)(\ell^{nc} - \ell^{c}) + \psi(e_1 + 1) - \psi(e_1 + \Delta \beta) + a$$
Rearranging the left hand side:

\[-\theta(1)\tilde{t}^{nc}(1 - \bar{e}_1) - \tilde{t}^e + \bar{t}^e \bar{e}_1 + \theta(1)\tilde{t}^e(1 - \bar{e}_1) + \theta(0)\tilde{t}^{nc}(1 - \xi_1) + \tilde{t}^e - \xi_1 \tilde{t}^e - \theta(0)\tilde{t}^e(1 - \xi_1)\]

That can be rewritten as:

\[\tilde{t}^e - \tilde{t}^e + \theta(1)(\tilde{t}^e - \tilde{t}^{nc})(1 - \bar{e}_1) + \theta(0)(\tilde{t}^{nc} - \tilde{t}^e)(1 - \xi_1) - \xi_1 \tilde{t}^e + \bar{t}^e \bar{e}_1\]

Hence,

\[\tilde{t}^e - \tilde{t}^e \geq \theta(1)(1 - \bar{e}_1)(\tilde{t}^{nc} - \tilde{t}^e) - \theta(0)(1 - \xi_1)(\tilde{t}^{nc} - \tilde{t}^e) + \psi(\bar{e}_1 + 1) - \psi(\xi_1 + \Delta \beta) + b\]
References


